APPLICATION OF QUADRATIC FORMULA

$Ax^2 + Bx + C = 0$

THEN $x = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

Ex SOLVE $(x-7)(x+2) = 4x-1$

$x^2 - 7x + 2x - 14 = 4x - 1$

$x^2 - 5x - 14 = 4x - 1$

$x^2 - 5x - 14 - 4x + 1 = 0$

$x^2 - 9x - 13 = 0$

$Ax^2 + Bx + C = 0$

$A = 1, \quad B = -9, \quad C = -13$

$x = \dfrac{-(-9) \pm \sqrt{(-9)^2 - 4 \times 1 \times (-13)}}{2 \times 1}$

$= \dfrac{9 \pm \sqrt{81 + 52}}{2}$

$= \dfrac{9 \pm \sqrt{133}}{2}$

$= \dfrac{9 \pm 11.53}{2}$

$= \dfrac{9 + 11.53}{2}$ (OR) $\dfrac{9 - 11.53}{2}$

$= \dfrac{20.53}{2}$ (OR) $-\dfrac{2.53}{2}$

$= 10.265$ (OR) -1.265

EXERCISE

(1) SOLVE $50E^2 + 150E - 100 = 0$

(2) SOLVE $x^2 + 2x - 5 = 0$

18 WEEK

1 ⟶ 3 PM

1 ⟶ 9 MATHS (1) E050 — CALCULUS + ALGEBRA

9 ⟶ 18 MATHS (2) E026 — DIFFERENTIAL EQUATION + LAPLACE TRANSFORM
MATHEMATICAL SERIES + MATRICE

3 ⟶ 4:30 PM

1 ⟶ 9 G048 CIRCUIT (1)

9 ⟶ 18 E025 CIRCUIT (2)

4:30 ⟶ 6 G047 + G049

POLYNOMIALS

$7x^{15} + 6x^{8} - 4x^{2}$

$13x^{2} - 6x - 8$

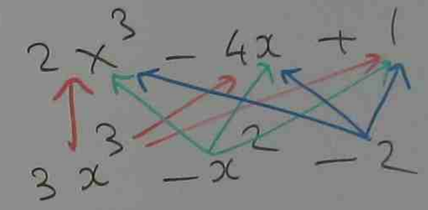
$5x + 3$ DEGREE 1

$4x^{3}$ DEGREE 3

$x^{m} \times x^{n} = x^{m+n}$

$\frac{x^{m}}{x^{n}} = x^{m-n}$

EX $(2x^{3} - 4x + 1) \times (3x^{3} - x^{2} - 2)$



$6x^{6} - 12x^{4} + 3x^{3}$

$-2x^{5} + 4x^{3} - x^{2}$

$-4x^{3} + 8x - 2$

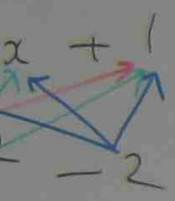
$6x^{6} - 2x^{5} - 12x^{4} + 3x^{3} - x^{2} + 8x - 2$

EX EXPAND

$(2x-3)(x^{2} - 4x + 2)$

FIND THE COEFFICIENT OF x^{3} IN EXPANSION

$+1)\times(3x^3-x^2-2)$



$x+1$

-2

$+3x^3$

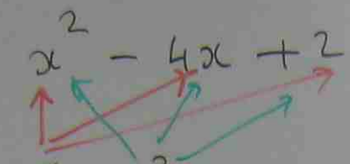
$+4x^3 \quad -x^2$

$-4x^3 \quad +8x \quad -2$

$x^4+3x^3-x^2+8x-2$

$x^2-4x+2)$

efficient of x^3 IN EXPANSION



x^2-4x+2

$2x-3$

$2x^3-8x^2+4x$

$-3x^2+12x-6$

$2x^3-11x^2+16x-6$

THE COEFFICIENT of x^3 IS 2

EX EVALUATE

$2x^2+7x-9$ WHEN $x=4$

$2(4)^2+7\times4-9$

$2\times16+28-9$

$32+28-9$

$60-9=51$

FACTORISATION

$6=2\times3$

$12=2\times6$

$12=4\times3$

$8ab+12ac$

$a(8b+12c)$

$a(4\times2b+4\times3c)$

$4a(2b+3c)$

EXERCISE

FACTORISE

(i) $4x+6$

(ii) $9-12a$

(iii) $ab+a$

(iv) $6xy-9x^2$

(v) $5x^2y-10ax^2+5x^2$

(vi) $3a^2b^2+6ab^2$

EX FACTORISE

$e^{-2x}\times e^x=$

e^x+e^{-2x}

$e^x(1+e^{-2x}$

EX $6e^{2x}$

$e^{2x}=e^{x+x}$

$6e^x\times e^x$

$e^x(6e^x$

$2e^x(3e^x$

FACTORISATION

$6 = 2 \times 3$

$12 = 2 \times 6$

$12 = 4 \times 3$

$8ab + 12ac$

$(8b + 12c)$

$(4 \times 2b + 4 \times 3c)$

$a(2b + 3c)$

EXERCISE

FACTORISE

i) $4x + 6$

ii) $9 - 12a$

iii) $ab + a$

iv) $6xy - 9x^2$

v) $5x^2y - 10ax^2 + 5x^2$

vi) $3a^2b^2 + 6ab^2$

EX FACTORISE $e^{x} + e^{-x}$

$e^{-2x} \times e^{x} = e^{-2x+x} = e^{-x}$

$e^{x} + e^{-2x} \times e^{x}$

$e^{x}(1 + e^{-2x})$ ✗✗

EX $6e^{2x} - 4e^{x}$

$e^{2x} = e^{x+x} = e^{x} \times e^{x}$

$6e^{x} \times e^{x} - 4e^{x}$

$e^{x}(6e^{x} - 4)$

$2e^{x}(3e^{x} - 2)$ ✗✗

EX $\dfrac{e^{x} + 3e^{3x}}{e^{-x} + 3e^{x}}$

$\dfrac{e^{x} + 3e^{2x+x}}{e^{-2x+x} + 3e^{x}}$

$\dfrac{e^{x} + 3e^{2x} \times e^{x}}{e^{-2x} \times e^{x} + 3e^{x}}$

$\dfrac{e^{x}(1 + 3e^{2x})}{e^{x}(e^{-2x} + 3}$

$\dfrac{1 + 3e^{2x}}{3 + e^{-2x}}$ ✗✗

$(a+b)\ (a-b)$

$a+b$
$a-b$

$a^2 + ab$
$\quad -ab - b^2$

$a^2 - b^2$

$a^2 - b^2 = (a+b)(a-b)$

$(x+y)(x+y)$

$x+y$
$x+y$

$x^2 + xy$
$\quad +xy + y^2$

$x^2 + 2xy + y^2$

$(x^2 + 2xy + y^2) = (x+y)(x+y)$

Ex FACTORISE

(a) $x^2 + 4x + 3$

(b) $\square^2 + 8\square + 12$

(c) $z^2 + 7z + 10$

(d) $x^2 + 8x + 15$

(e) $E^2 + 11E + 24$

(a) $x^2 + 4x + 3$

x 3
x 1

$3x + x = 4x$

$(x^2 + 4x + 3) = (x+3)(x+1)$

(b) $\square^2 +$

$\square$
$\square$

$2\square +$

$(\square^2 + 8\square$

(c) $z^2 + 7$

z
z

$2z +$

$(z^2 + 7$

CTORISE

$+4x+3$

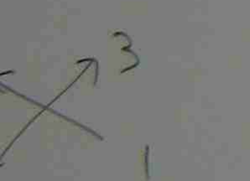
$+8\theta+12$

$+7z+10$

$^2+8x+15$

$^2+11E+24$

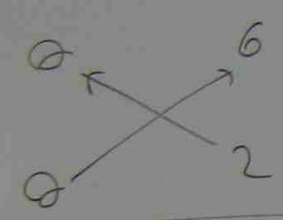
$+4x+3$



$x+x=4x$

$+3)=(x+3)(x+1)$

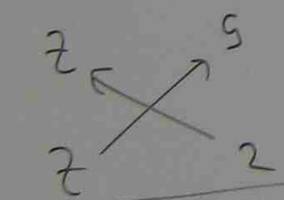
(b) $\theta^2+8\theta+12$



$2\theta+6\theta=8\theta$

$(\theta^2+8\theta+12)=(\theta+6)(\theta+2)$

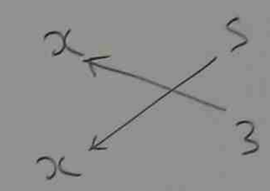
(c) $z^2+7z+10$



$2z+5z=7z$

$(z^2+7z+10)=(z+5)(z+2)$

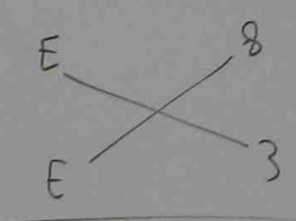
(d) $x^2+8x+15$



$3x+5x=8x$

$(x^2+8x+15)=(x+5)(x+3)$

(e) $E^2+11E+24$



$3E+8E=11E$

$E^2+11E+24=(E+8)(E+3)$

EXERCISE

FACTORISE

(a) $f^2+17f+16$

(h) $\theta^2+12\theta+32$

POLY NO

$7x^{15}+$

$13x^2-$

$5x+$

$4x^3$

$x^m \times x^n$

$\frac{x^m}{x^n}$

$(a+b)(a-b)$

$a+b$

$a-b$

a^2+ab

$-ab-b^2$

a^2-b^2

$a^2-b^2=(a+b)(a-b)$

$(x+y)(x+y)$

$x+y$

$x+y$

x^2+xy

$+xy+y^2$

$x^2+2xy+y^2$

$(x^2+2xy+y^2)=(x+y)(x+y)$

Ex FACTORISE

(a) x^2+4x+3

(b) $\Theta^2+8\Theta+12$

(c) $z^2+7z+10$

(d) $x^2+8x+15$

(e) $E^2+11E+24$

(a) x^2+4x+3

x 3

x 1

$3x+x=4x$

$(x^2+4x+3)=(x+3)(x+1)$

(b)

(c) z

z

z

2

$(z^2$

ACTORISE

$+4x+3$

$+8\theta+12$

$+7z+10$

$^2+8x+15$

$^2+11E+24$

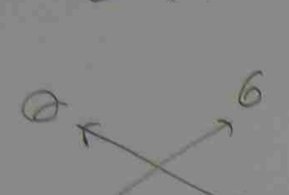
$+4x+3$



$x+x=4x$

$x+3)=(x+3)(x+1)$

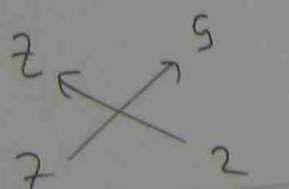
(b) $\theta^2+8\theta+12$



$2\theta+6\theta=8\theta$

$(\theta^2+8\theta+12)=(\theta+6)(\theta+2)$

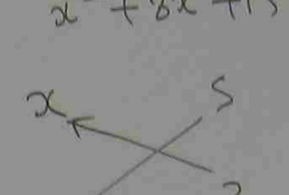
(c) $z^2+7z+10$



$2z+5z=7z$

$(z^2+7z+10)=(z+5)(z+2)$

(d) $x^2+8x+15$



$3x+5x=8x$

$(x^2+8x+15)=(x+5)(x+3)$

(e) $E^2+11E+24$



$3E+8E=11E$

$E^2+11E+24=(E+8)(E+3)$

EXERCISE

FACTORISE

(a) $f^2+17f+16$

(h) $\theta^2+12\theta+32$

Ex

x

$2x+3x$

x^2-9

8

4

EX SOLVE $\dfrac{8x^2+40x+48}{4x^2-36}$

$$\frac{8(x^2+5x+6)}{4(x^2-9)}$$

$x \quad 3$

$x \quad 2$

$2x+3x=5x$

$(x+3)(x+2)$

$x^2-9 = (x-3)(x+3)$

$$\frac{8(x+3)(x+2)}{4(x-3)(x+3)}$$

$$\frac{8(x+2)}{4(x-3)}$$

$$\frac{2(x+2)}{}$$

EX

SOLVE (a) $\dfrac{6-2k}{3} = \dfrac{2(k-7)}{k}$

(b) $\dfrac{x-4}{3} = \dfrac{5-2x}{x-2}$

(c) $\dfrac{5}{R+3} = \dfrac{3+R}{5}$

(a) $\dfrac{6-2k}{3} = \dfrac{2(k-7)}{k}$

$k(6-2k) = 3\times 2(k-7)$

$6k-2k^2 = 6(k-7)$

$6k-2k^2 = 6k-42$

$-2k^2 = -42$

$k^2 = 21$

$k = \sqrt{21}$

$= \frac{2(k-7)}{k}$

$= \frac{5-2x}{x-2}$

$\frac{3+R}{5}$

k

$(k-7)$

$k-7)$

-42

2

21

(b) $\frac{x-4}{3} = \frac{5-2x}{x-2}$

$(x-4)(x-2) = 3(5-2x)$

$x^2 - 4x - 2x + 8 = 15 - 6x$

$x^2 - 6x + 8 = 15 - 6x$

$x^2 - 6x + 6x = 15 - 8$

$x^2 = 7$

$x = \sqrt{7}$

(c) $\frac{9R}{R+3} = \frac{3+R}{5}$

$5 \times 5 = (R+3)(R+3)$

$25 = R^2 + 3R + 3R + 9$

$25 = R^2 + 6R + 9$

$R^2 + 6R - 16 = 0$

$R \quad +8$

$R \quad -2$

$8R - 2R = 6R$

$(R+8)(R-2) = 0$

$R + 8 = 0 \text{ (OR) } R - 2 = 0$

$\therefore R = -8 \text{ (OR) } R = 2$

APPLICA TIO

$A x^2 + 13x$

THEN

ET

Sol

$(x -$

$x^2 -$

$x^2 -$

$x^2 - 5x$

$x^2 - 9$

$Ax^2 +$

$A = 1$

APPLICATION OF QUADRATIC FORMULA

$$Ax^2 + Bx + C = 0$$

THEN $$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

EX — SOLVE

$$(x-7)(x+2) = 4x-1$$

$$x^2 - 7x + 2x - 14 = 4x - 1$$

$$x^2 - 5x - 14 = 4x - 1$$

$$x^2 - 5x - 14 - 4x + 1 = 0$$

$$x^2 - 9x - 13 = 0$$

$$Ax^2 + Bx + C = 0$$

$$A = 1, \quad B = -9, \quad C = -13$$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4 \times 1 \times (-13)}}{2 \times 1}$$

$$= \frac{9 \pm \sqrt{81 + 52}}{2}$$

$$= \frac{9 \pm \sqrt{133}}{2}$$

$$= \frac{9 \pm 11.53}{2}$$

$$= \frac{9 + 11.53}{2} \text{ (OR) } \frac{9 - 11.53}{2}$$

$$= \frac{20.53}{2} \text{ (OR) } -\frac{2.53}{2}$$

$$= 10.265 \text{ (OR) } -1.265$$

EXERCISE

(1) SOLVE $50E^2 + 150E - 100 = 0$

(2) SOLVE $x^2 + 2x - 5 = 0$

FUNCTIONS AND THEIR GRAPHS

$$f(x) = x - 3$$

If $x=1 \longrightarrow f(1) = 1-3 = -2$

$x=2 \longrightarrow f(2) = 2-3 = -1$

$x=3 \longrightarrow f(3) = 3-3 = 0$

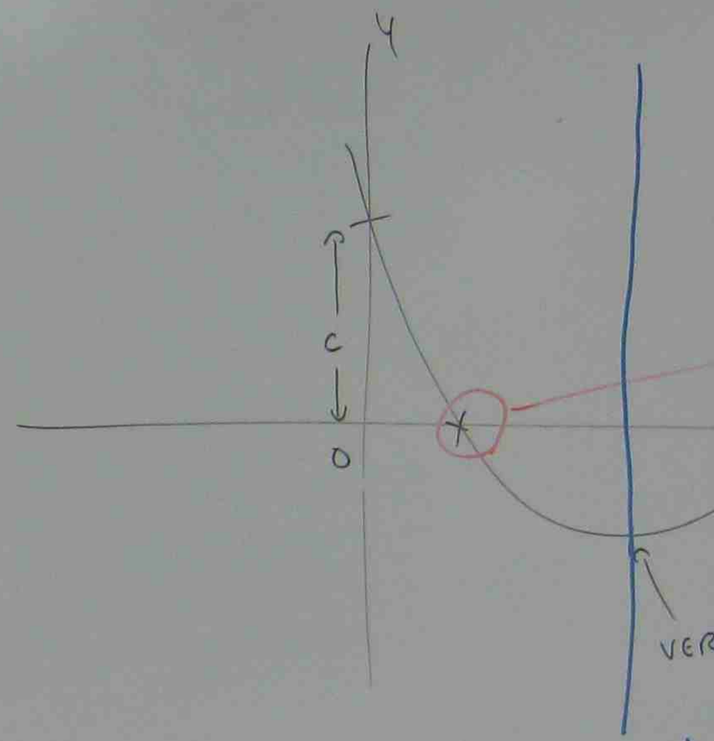
$$g(x) = 4 + x^2$$

If $x=1 \longrightarrow g(1) = 4+1^2 = 5$

$x=2 \longrightarrow g(2) = 4+2^2 = 8$

$x=3 \longrightarrow g(3) = 4+3^2 = 13$

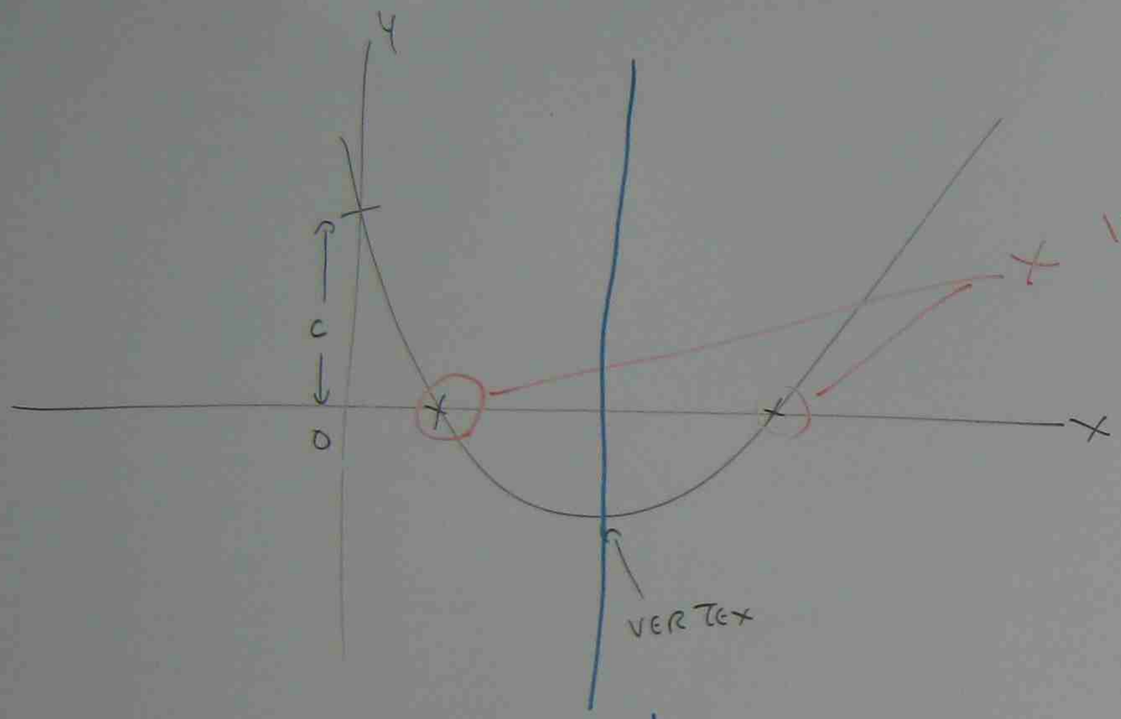
PLOTTING THE GRAPH FOR EQUATION



$X = \frac{-b}{2}$

(AXIS

PLOTTING THE GRAPH FOR EQUATION $y = ax^2 + bx + c$



$X = \frac{-b}{2a}$

(AXIS OF SYMMETRY)

STEP(1)

Plot $y = 2x^2 - 12x + 13$

COMPARE $ax^2 + bx + c$

$\therefore a = 2, \quad b = -12, \quad c = 13$

FIND AXIS of SYMMETRY

$$\text{AXIS of SYMMETRY} = \frac{-b}{2a} = \frac{-(-12)}{2 \times 2} = 3$$

Y VALUE AT AXIS of SYMMETRY

$y = 2x^2 - 12x + 3$

$y = 2(3)^2 - 12 \times 3 + 3$

$= -5$

$X = 3$ (AXIS OF SYMMETRY)

VERTEX IS LOCATED AT $y = -5$

$c = 13$

$= 3$

$+3$

3

$)$

STEP (2) FIND X AXIS CROSSINGS

$y = 0$

FIND X VALUE

$y = 2x^2 - 12x + 13$

$2x^2 - 12x + 13 = 0$

$Ax^2 + Bx + C = 0$

$A = 2, \quad B = -12, \quad C = 13$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

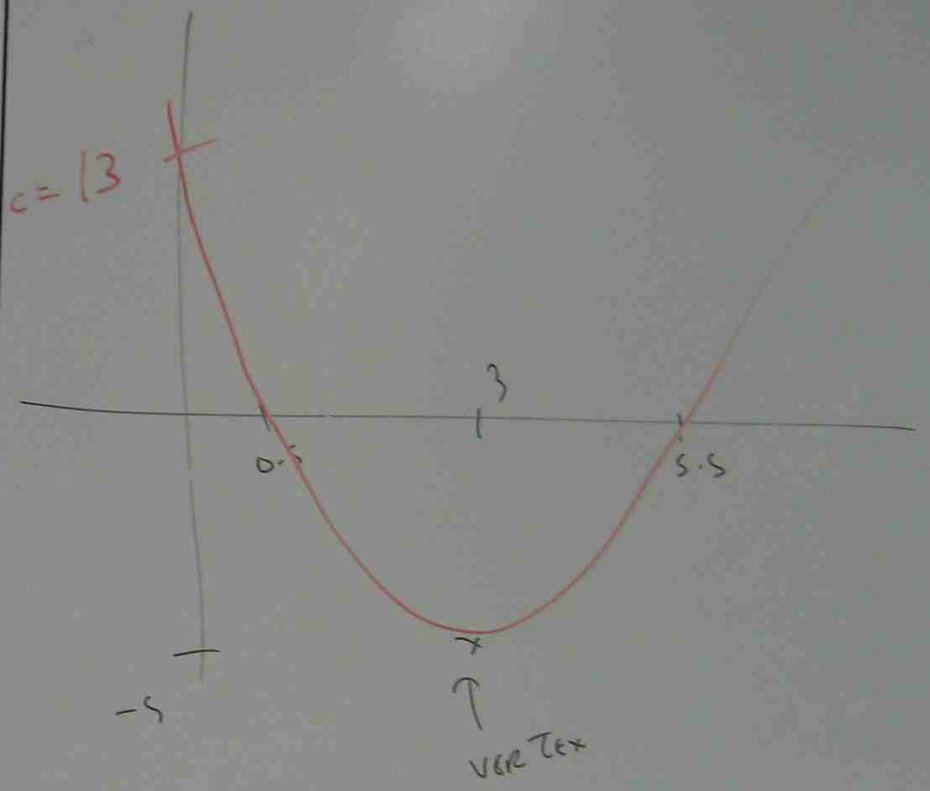
$$= \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \times 2 \times 13}}{2 \times 2}$$

$$= \frac{12 \pm \sqrt{100}}{4}$$

$$= \frac{12 \pm 10}{4}$$

$$= \frac{12+10}{4} \text{ (OR) } \frac{12-10}{4}$$

$= 5.5$ (OR) 0.5



(b) $x -$

$(x - 4$

$x^2 - 4$

$x^2 - 6x$

$x^2 =$

$x =$

(c)

$R =$

$5 \times$

$2($

(b) $\frac{x-4}{3} = \frac{5-2x}{x-2}$

$(x-4)(x-2) = 3(5-2x)$

$x^2 - 4x - 2x + 8 = 15 - 6x$

$x^2 - 6x + 8 = 15 - 6x$

$x^2 - 6x + 6x = 15 - 8$

$x^2 = 7$

$x = \sqrt{7}$

(c) $\frac{5}{R+3} = \frac{3+R}{5}$

$5 \times 5 = (R+3)(R+3)$

$25 = R^2 + 3R + 3R + 9$

$25 = R^2 + 6R + 9$

$R^2 + 6R - 16 = 0$

R + 8

R − 2

$8R - 2R = 6R$

$(R+8)(R-2) = 0$

$R + 8 = 0$ (OR) $R - 2 = 0$

$\therefore R = -8$ (OR) $R = 2$

APPLICATION OF

$Ax^2 + Bx + C =$

THEN $x =$

E+ SOLVE

$(x - 7)$ (

$x^2 - 7x +$

$x^2 - 5x -$

$x^2 - 5x - 14$

$x^2 - 9x - 1$

$Ax^2 + Bx +$

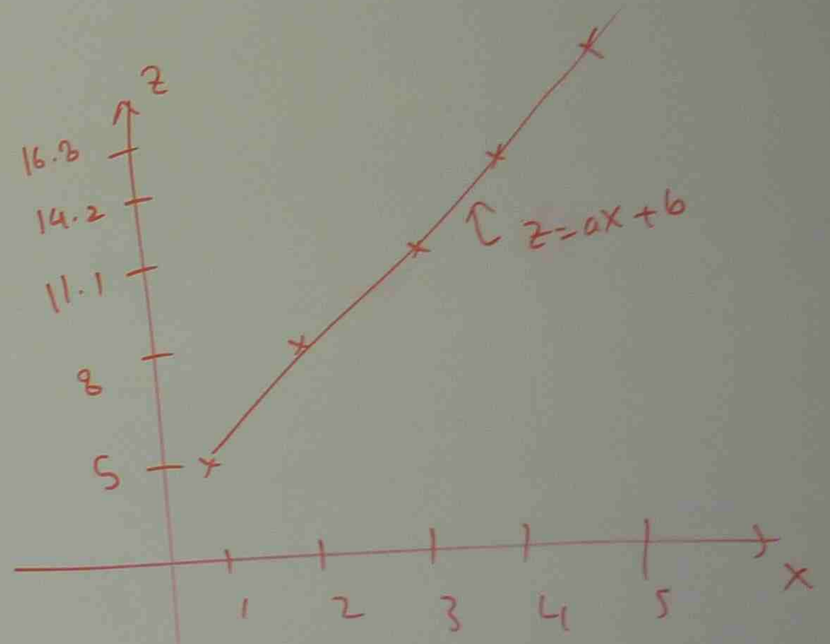
$A = 1$, B

pb THE FOLLOWING VALUES of X AND Y ARE BELIEVED TO SATISFY THE EQUATION OF $y = ax^2 + bx$. FIND A LINEAR EQUATION THAT SUITS THIS INFORMATION AND SO EVALUATE a & b.

X	1	2	3	4	5
Y	5	16	34	57	84

$$z = \frac{y}{x} = \frac{ax^2 + bx}{x} = ax + b$$

X	1	2	3	4	5
$z = \frac{Y}{X}$	$5/1 = 5$	$16/2 = 8$	$\frac{34}{3} = 11.3$	$\frac{57}{4} = 14.2$	$\frac{84}{5} = 16.8$





LOGARITHMS AND EXPONENTIAL EQUATION

EXPONENTIAL FORM 2^{-3}, 5^{-1}, a^2, a^3

$$\text{Log}_b N = L \implies N = b^L$$

Ex WRITE EXPONENTIAL FORM

$$\log_3 9 = 2 \longrightarrow 9 = 3^2$$

Ex WRITE THE LOGARITHMIC FORM

$$5^2 = 25$$

$$2 = \log_5 25$$

EVALUATE $\log_3$

3	81
3	27
3	9
	3

$\log_3 3^4$

$\log_A A = 1$

ent

tafensw.edu.au

EN TIAL EQUATION

2^{-3}, 5^{-1}, a^2, a^3

$\Rightarrow$ $N = b^L$

orm

$q = 3^2$

form

EVALUATE $\log_3 81 = x$

3 | 81

3 | 27

3 | 9

| 3

$3^4 = 81$

$\log_3 3^4 = 4 \log_3 3$

$= 4 \times 1$

$= 4$

$\boxed{\log_A A = 1}$

EVALUATE

$\log_9 \frac{1}{3}$

$\log_9 \frac{1}{3} = x$

$\frac{1}{3} = 9^x$

$\frac{-1}{3} = \left(\frac{2}{3}\right)^x$

$\frac{-1}{3} = 3^{2x}$

$2x = -1$

$x = -\frac{1}{2}$

Ex

$\log_5 \sqrt[3]{5} =$

$\log_5 \sqrt[3]{5} =$

$\sqrt[3]{5} =$

$x = 1/3$

DECIBEL

RATIO OF

10:1

100:

1000

10^4

10^5

Ex

$\log_5 \sqrt[3]{5} = ?$

$\log_5 \sqrt[3]{5} = x$

$\sqrt[3]{5} = 5^x$

$5^{1/3} = 5^x$

$x = 1/3$

$$\text{NO_OF BEL} = \log_{10} \frac{P_2}{P_1}$$

P_1 = IN PUT POWER

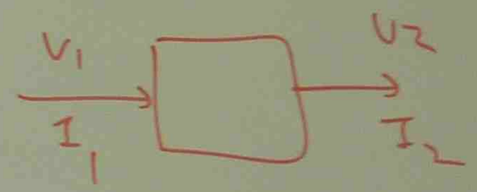
P_2 = OUT PUT POWER

1 BEL = 10 DECIBEL

$$\text{NO. OF DECIBEL} = 10 \log_{10} \frac{P_2}{P_1}$$

DECIBEL

RATIO OF POWER	RATIO EXPRESSED IN BEL	DECIBEL
10 : 1	1 BEL (1B) =	10 dB
100 : 1	2 BEL (2B) =	20dB
1000 : 1	3 BEL (3B) =	30dB
10^4 : 1	4 BEL (4B)	= 40dB
10^5 : 1	5 BEL (5B)	= 50dB



$$dB = 20 \log \frac{V_2}{V_1}$$

$$dB = 20 \log_{10} \frac{I_2}{I_1}$$

Ex IF A

OUT PU

$dB =$

$=$

Ex IF AN AMPLIFIER HAS AN INPUT POWER of 0.0013 WATT OUTPUT of 6.5 WATT. CALCULATE POWER GAIN IN dB

$$dB = 10 \log_{10} \frac{P_2}{P_1}$$

$$= 10 \log_{10} \frac{6.5}{0.0013}$$

$$= 10 \log_{10} 5000$$

$$= 10 \times 3.7$$

$$= 37 \text{ dB}$$

Ex (a) AN AMPLIFIER HAS INPUT POWER 1.7 mW, OUTPUT POWER 5.8 WATT. FIND POWER GAIN.

(b) AN ATTENUATOR HAS INPUT POWER 3.6 WATT, OUTPUT POWER 4.5 mW. FIND ATTENUATION.

(a) POWER GAIN (dB) $= 10 \log_{10} \frac{P_2}{P_1}$

$$= 10 \log_{10} \frac{5.8}{1.7 \times 10^{-3}}$$

$$= 10 \times 3.53 = 35.3 \text{ dB}$$

(b) POWER ATTENUATION (dB) $= 10 \log_{10} \frac{P_2}{P_1}$

$$= 10 \log_{10} \frac{4.5 \times 10^{-3}}{3.6}$$

$$= 10 \log_{10} 1.25 \times 10^{-3}$$

$$= -29 \text{ dB}$$

THE
FIRST
SECOND
THIRD
EVA

3 WATT

dB

Ex AN AMPLIFIER HAS INPUT POWER 1.7 mW,

(a) OUTPUT POWER 5.8 WATT . FIND POWER GAIN.

(b) AN ATTENUATOR HAS INPUT POWER 3.6WATT, OUTPUT POWER 4.5 mW . FIND ATTENUATION.

(a) POWER GAIN (dB) $= 10 \log_{10} \frac{P_2}{P_1}$

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$= 10 \log_{10} \frac{4.5 \times 10^{-3}}{3.6}$

$= 10 \log_{10} 1.25 \times 10^{-3}$

$= -29$ dB

THREE LAWS of LOGARITHM

FIRST $\log(A \times B) = \log A + \log B$

SECOND $\log \frac{A}{B} = \log A - \log B$

THIRD $\log A^n = n \log A$

EVALUATE $\log_3 36 - 2\log_3 2$

$\log_3 36 - \log_3 2^2$

$\log_3 36 - \log_3 4$

$\log_3 \frac{36}{4}$

$\log_3 9 = \log_3 3^2 = 2\log_3 3$

$= 2 \times 1$

$= 2$

CHANGE OF BASE

$$\log_a N = \frac{\log_b N}{\log_b a}$$

Solution of Logarithmic Equation

Ex(1) $\log_{10} x = -2$ FIND x

$$x = 10^{-2} = \frac{1}{100} = 0.01$$

Ex(2) $\log_2 (0.5v + 0.2) = -2$

FIND v

$$\log_2 (0.5v + 0.2) = -2$$

$$0.5v + 0.2 = 2^{-2}$$

$$0.5v + 0.2 = \frac{1}{2^2}$$

$$0.5v + 0.2 = \frac{1}{4}$$

$$0.5v + 0.2 = 0.25$$

$$0.5v = 0.05$$

$$v = \frac{0.05}{0.5}$$

$$= 0.1$$

Ex(3) SOLVE $2^x + 5 = 13$

$$2^x = 13 - 5 = 8$$

$$2^x = 2^3$$

$$x = 3$$

Ex(4) $\log_{10} \frac{k}{k-x} = t$

FIND x

$$\frac{k}{k-x} = 10^t$$

$$\frac{k}{10^t} = k - x$$

$$x = k - \frac{k}{10^t} = k\left(1 - \frac{1}{10^t}\right) = k\frac{(10^t - 1)}{10^t}$$

Ex PLOT THE

x VS

$x = 0$

$y = \log$

$2^x + 5 = 13$

$13 - 5 = 8$

2^3

3

$\frac{k}{k-x} = t$

10

FIND X

$= 10^t$

$= k - X$

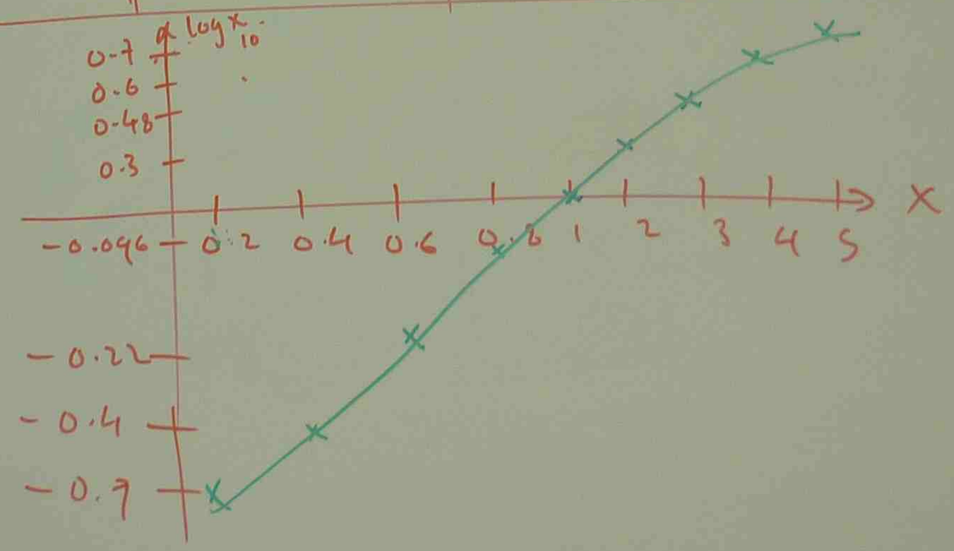
$k - \frac{k}{10^t} = k\left(1 - \frac{1}{10^t}\right) = \frac{k(10^t - 1)}{10^t}$

Ex PLOT THE FOLLOWING GRAPH

X VS $Y = \log_{10} X$ FOR THE FOLLOWING X VALUES

$X = 0.2, 0.4, 0.6, 0.8, 1, 2, 3, 4, 5$

X	0.2	0.4	0.6	0.8	1	2	3	4	5
$Y = \log_{10} X$	−0.7	−0.4	−0.22	−0.096	0	0.3	0.48	0.6	0.7



EXERCISE

FIND (a) Lo

(b) USE

Lo

(c) AN

AND

F

(d) FI

EXERCISE

FIND (a) $\log_3 \frac{1}{3^{15}}$

(b) USE CALCULATOR AND FIND THE VALUE

$$\log_{10}\sqrt{563} - \log_{10}\sqrt{107}$$

(c) AN AMPLIFIER HAS 10mW INPUT POWER AND 40W OUTPUT POWER FIND GAIN IN dB

(d) FIND (k) $3^{k+1} = 5$

NON LINEAR EMPIRICAL EQUATIONS

CONVERSION TO LINEAR FORM

$$y = ax^2 + bx$$

TO CHANGE TO LINEAR FORM, DIVIDE BY X

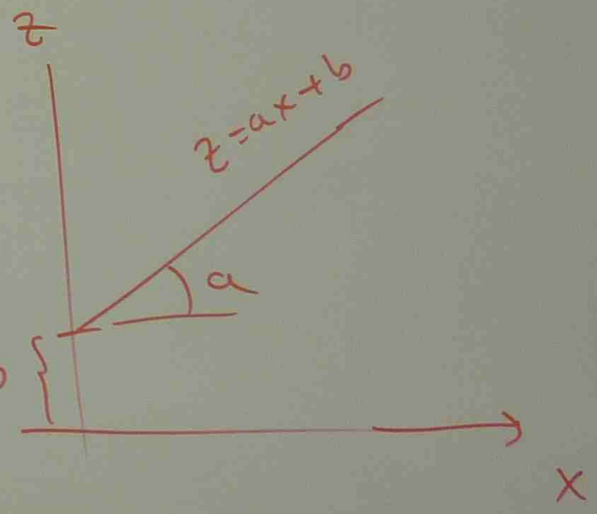
$$\frac{y}{x} = \frac{ax^2}{x} + \frac{bx}{x}$$

$$\frac{y}{x} = ax + b$$

IF $\frac{y}{x} = z$

THEN $\boxed{z = ax + b}$

LINEAR FORM



pb

SATIS

EQUA

a

X	
Y	

z=

X

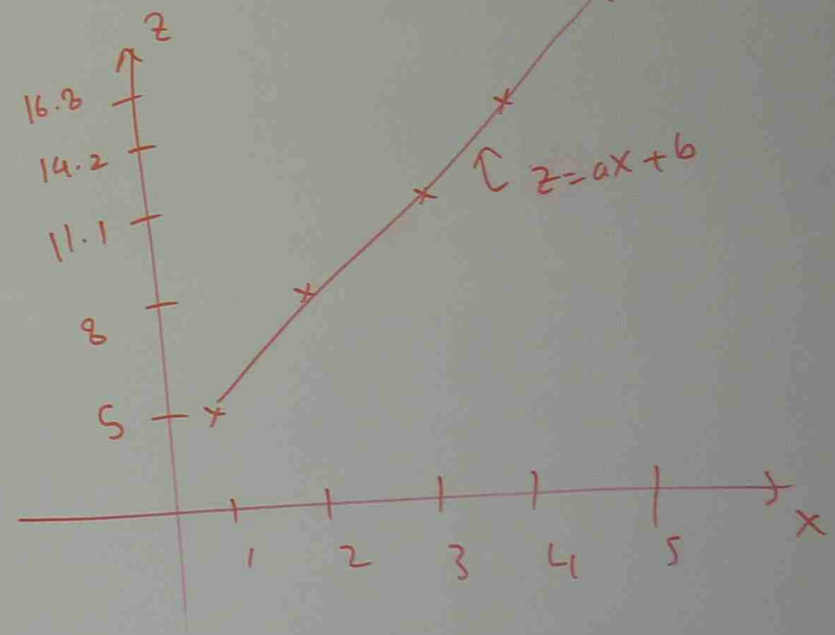
z= Y/X

P1) THE FOLLOWING VALUES of x AND y ARE BELIEVED TO SATISFY THE EQUATION OF $y = ax^2 + bx$. FIND A LINEAR EQUATION THAT SUITS THIS INFORMATION AND SO EVALUATE a & b.

x	1	2	3	4	5
y	5	16	34	57	84

$$z = \frac{y}{x} = \frac{ax^2 + bx}{x} = ax + b$$

x	1	2	3	4	5
$z = \frac{y}{x}$	$5/1 = 5$	$16/2 = 8$	$\frac{34}{3} = 11.1$	$\frac{57}{4} = 14.2$	$\frac{84}{5} = 16.8$



x

SOLUTION OF LOGARITHMIC EQUATION

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$$0.5V + 0.2 = \frac{1}{4}$$

$$0.5V + 0.2 = 0.25$$

$$0.5V = 0.05$$

$$V = \frac{0.05}{0.5}$$

$$= 0.1$$

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$$x = k - \frac{k}{10^t} = k\left(1 - \frac{1}{10^t}\right) = k\left(\frac{10^t - 1}{10^t}\right)$$

Ex PLOT THE FO

X VS

$x = 0.2$

$y = \log_{10}$

E $2^x + 5 = 13$

$= 13 - 5 = 8$

$= 2^3$

$= 3$

$\log_{10} \frac{k}{k-x} = t$

FIND x

$\frac{k}{k-x} = 10^t$

$\frac{k}{k-x} = 10^t$

$k - x = \frac{k}{10^t}$

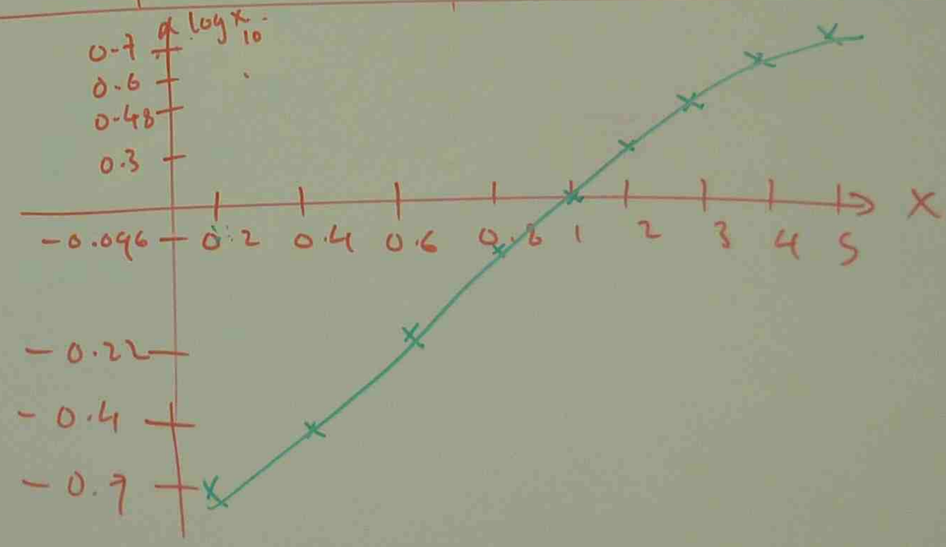
$x = k - \frac{k}{10^t} = k\left(1 - \frac{1}{10^t}\right) = \frac{k(10^t - 1)}{10^t}$

Ex PLOT THE FOLLOWING GRAPH

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$y = \log_{10} x$	−0.7	−0.4	−0.22	−0.096	0	0.3	0.48	0.6	0.7



EXERCISE

FIND (a)

(b)

(c)

(d)

5
0.7

EXERCISE

FIND (a) $\log_3 \frac{1}{3^{15}}$

(b) USE CALCULATOR AND FIND THE VALUE

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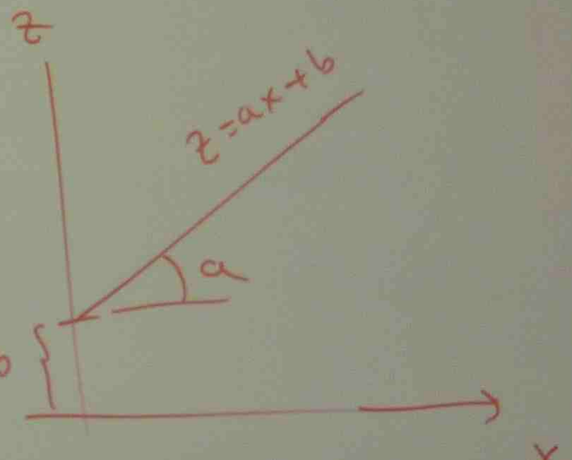
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$$\frac{y}{x} = ax + b$$

IF $\frac{y}{x} = z$

THEN $\boxed{z = ax + b}$

LINEAR FORM



ph

SAT

EQU

a

X
Y

z

z=

ADIAN CONVERSION

π RADIAN

$\frac{\pi}{2}$ RADIAN

$\frac{\pi}{2}$ RADIAN

FOLLOWING ANGLES INTO

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{10}$ (e) $\frac{5\pi}{12}$

$= 45^\circ$ (c) $\frac{\pi}{2} \to \frac{180}{2} = 90^\circ$ (e) $\frac{5\pi}{12} \to \frac{5 \times 180}{12}$

$= 75^\circ$

(d) $\frac{\pi}{10} \to \frac{180}{10} = 18^\circ$

Ex. CONVERT THE FOLLOWINGS IN TO RADIAN

(a) 45° (b) 210° (c) 120°

(a) $180^\circ \longrightarrow \pi$

$45 \longrightarrow ? = \frac{\pi \times 45}{180} = \frac{\pi}{4}$

(b) $180 \to \pi$

$210 \longrightarrow ? = \frac{\pi \times 210}{180} = \frac{7\pi}{6}$

(c) $180 \to \pi$

$120 \longrightarrow ? = \frac{\pi \times 120}{180} = \frac{2\pi}{3}$

SINE

0

COS

0

(c)

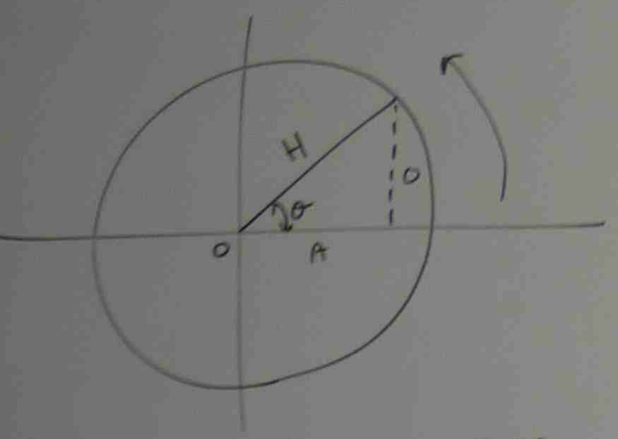
8

(d) $\pi = 1$

$\frac{\pi}{4} \Rightarrow$

10

CIRCULAR FUNCTIONS



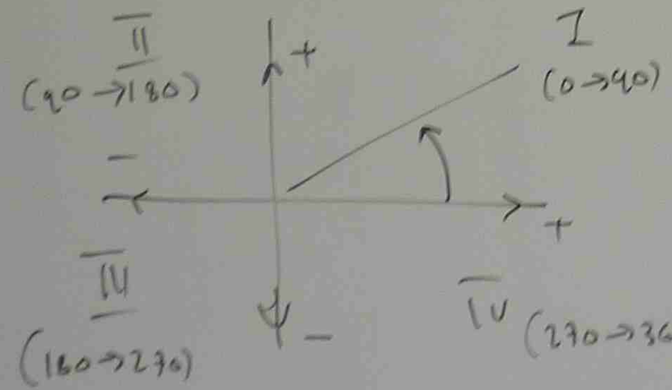
A - ADJACENT SIDE
O - OPPOSITE SIDE
H - HYPOTENUSE

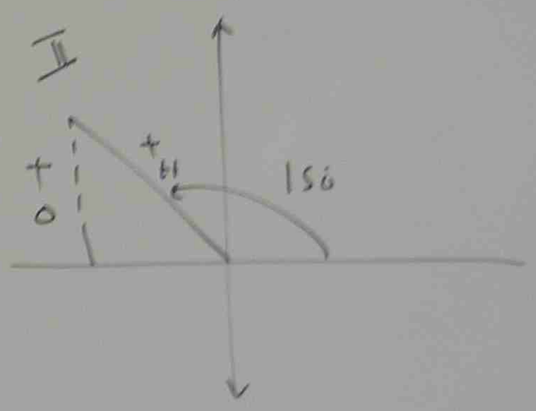
$\sin\theta = \frac{\text{OPPOSITE SIDE } (O)}{\text{HYPOTENUSE } (H)}$

sin	cos	tan	cot	sec	cosec
O	A	O	A	H	H
H	H	A	O	A	O

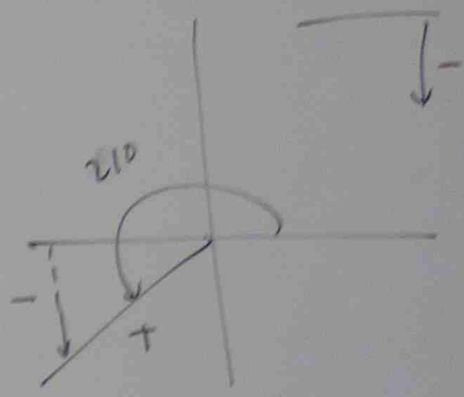


$\sin 45 = +0.707$

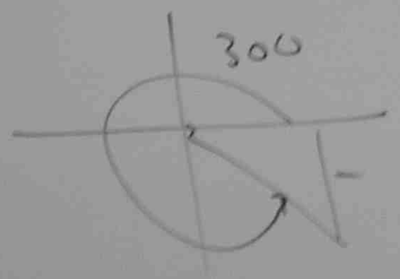




$\sin 150 = +0.5$

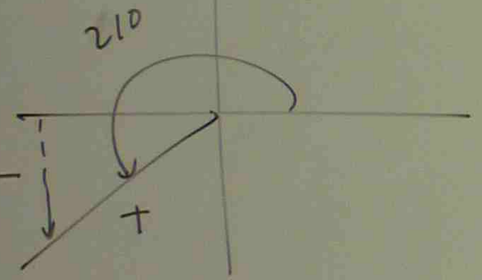


$\sin 210 = -0.5$



$\sin 300 = -0.866$

+ +



$\sin 210 = -0.5$

$\cos 0 = 1$

$\cos 90 = 0$

$\cos 180 = -1$

$\cos 270 = 0$

$\cos 360 = 1$

$\sin 0 = 0$

$\sin 90 = 1$

$\sin 180 = 0$

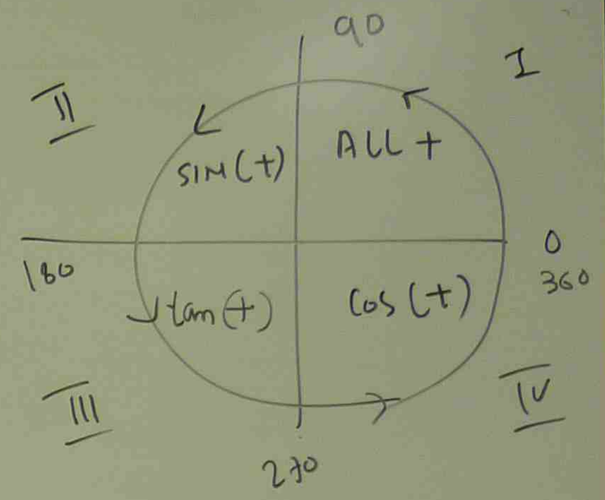
$\sin 270 = -1$

$\sin 360 = 0$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\tan 0 = \frac{\sin 0}{\cos 0} = \frac{0}{1} = 0$$

$$\tan 90 = \frac{\sin 90}{\cos 90} = \frac{1}{0} = \infty$$



EXERCISE

USE CALCULATOR AND FIND THE VALUES OF THE FOLLOWING ANGLES

tan 130 , tan 230, tan 180

sin 50 , sin 315

cos 120 , cos 230

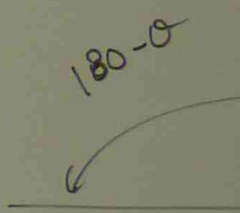
cos 315

$\sin\theta =$

$\cos\theta =$

$\tan\theta =$

SUPPLEMENT



sin (18

cos (180

tan (180

10

I

LL +

0
360

os (+)

IV

AND FIND
THE FOLLOWING

0, tan180

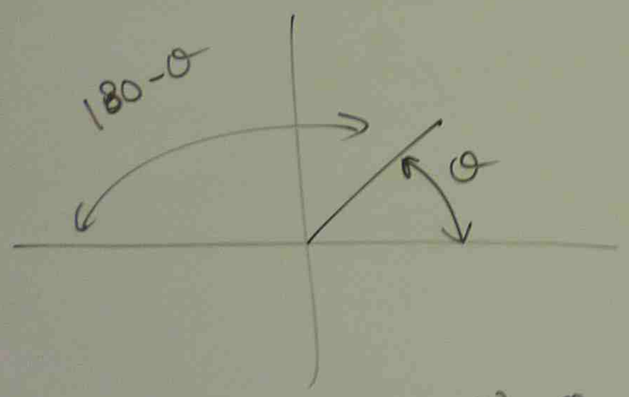
-30

$$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$$

$$\cos\theta = \frac{1}{\sec\theta}$$

$$\tan\theta = \frac{1}{\cot\theta}$$

SUPPLEMENTARY ANGLES FOR 180°



$$\sin(180-\theta) = \sin\theta$$

$$\cos(180-\theta) = -\cos\theta$$

$$\tan(180-\theta) = -\tan\theta$$

NEGATIVE ANGLES

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

THE TRIGO RATIO FOR 90° SUPPLEMENTARY ANGLES

$$\sin(90-\theta) = +\cos\theta$$

$$\cos(90-\theta) = +\sin\theta$$

$$\sin(90+\theta) = +\cos\theta$$

$$\cos(90+\theta) = -\sin\theta$$

Ex. PROVE THA

(a) sin(90-

$\therefore \cos\theta =$

FURTHER cos 80° =

(b) Sin

Sin

EMENTARY

Ex PROVE THAT (a) $\sin 10^\circ = \cos 80^\circ$

(b) $\operatorname{cosec} 85 = \sec 5$

(a) $\sin(90-\theta) = \cos\theta$

$\cos\theta = \sin(90-\theta)$

FOR $\cos 80^\circ = \sin(90-80)$

$= \sin 10^\circ$

(b) $\sin(90-\theta) = \cos\theta$

$\theta = 5^\circ$

$\sin(90-5) = \cos 5$

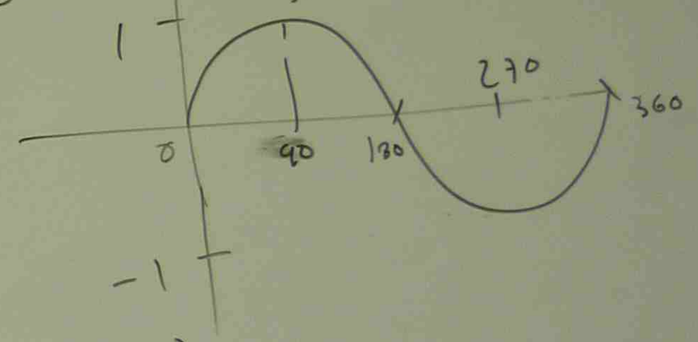
$\sin 85 = \cos 5$

$\frac{1}{\operatorname{cosec} 85} = \frac{1}{\sec 5}$

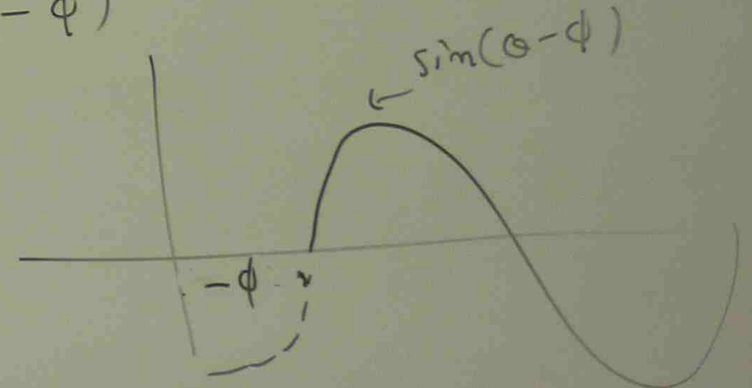
$\sec 5 = \operatorname{cosec} 85$

PLOTTING GRAPHS FOR TRIGONOMETRIC ANGLES

$\sin\theta$



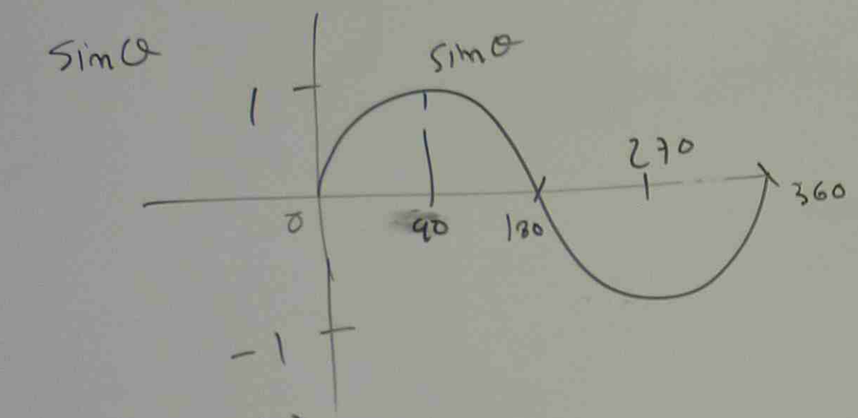
$\sin(\theta-\phi)$

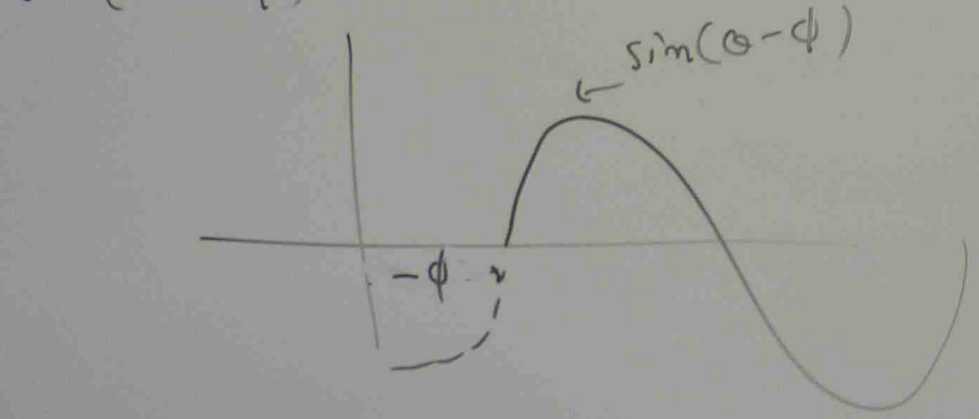


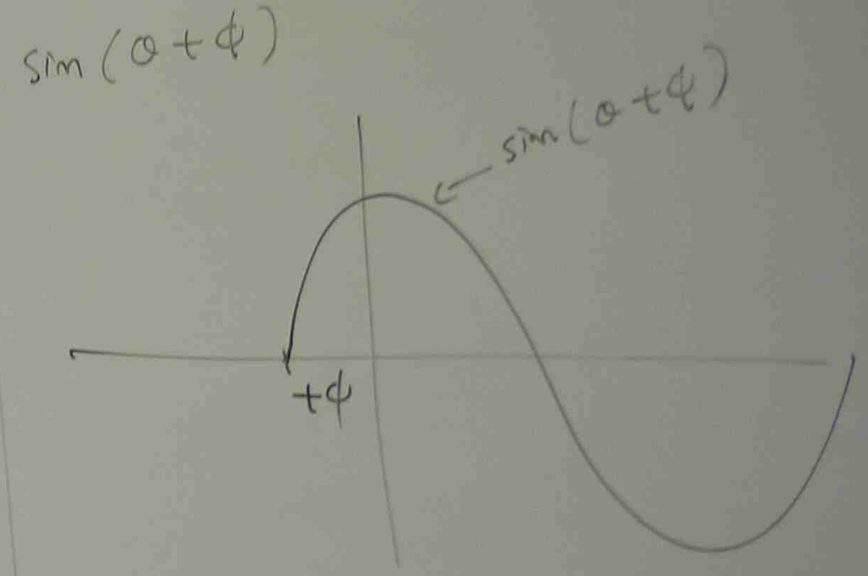
$\sin(\theta+\phi$

$\sec S = 8 \operatorname{cosec} 8S$

PLOTTING GRAPHS FOR TRIGONOMETRIC ANGLES:







EX. PLOT THE FOLLOWING GRAPHS

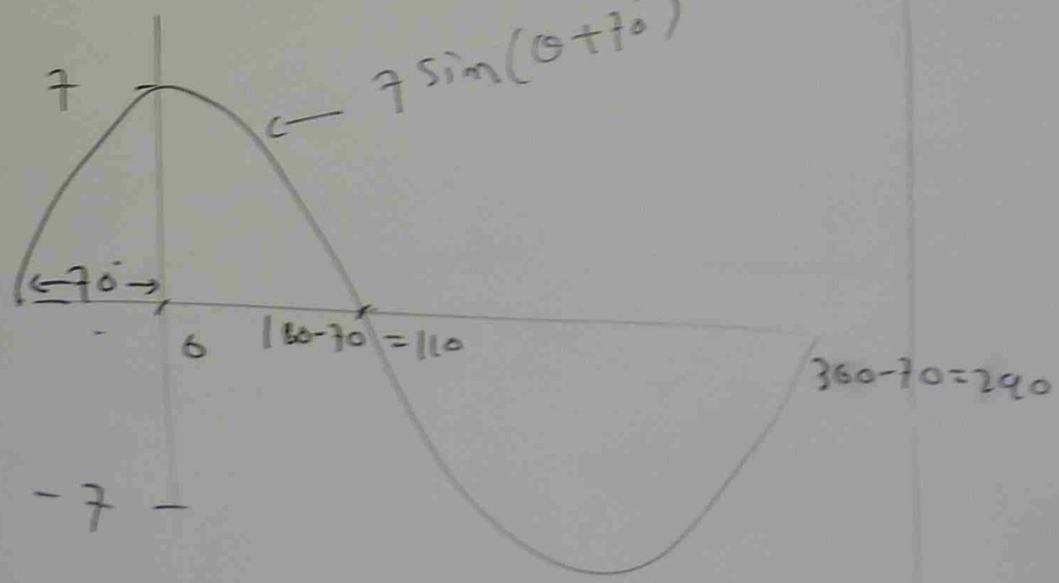
(a) $7 \sin(\theta + 70^\circ)$

(b) $50 \cos(\theta - 40)$

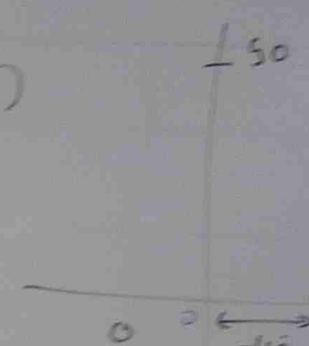
(c) $8 \cos\left(\theta - \frac{\pi}{3}\right)$

(d) $10 \sin\left(\theta + \frac{\pi}{4}\right)$

(a)



(b)



(c)

$\pi \rightarrow 180$

$\frac{\pi}{3} \rightarrow 1$

$8 \cos(\theta - \pi/3)$

↑ Amplitude

ING GRAPHS

)

o)

)

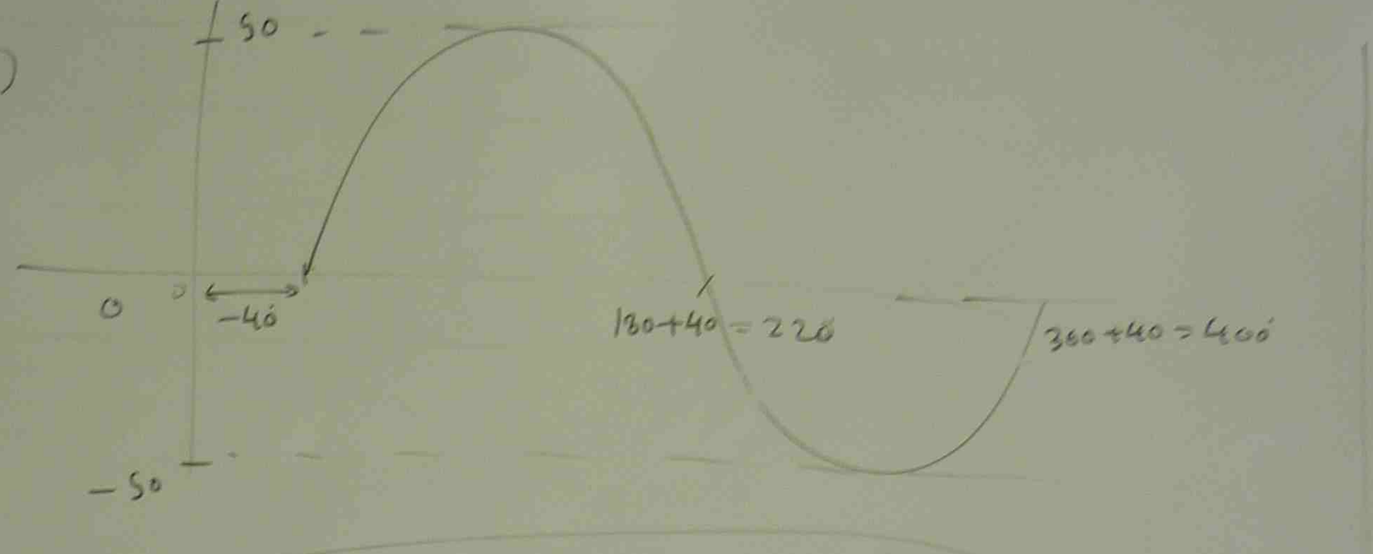
$\frac{\pi}{4}$)

$7\sin(\theta+70^\circ)$

=110

360-70=290

(b)



(c) DEGREE ⟷ RADIAN CONVERSION

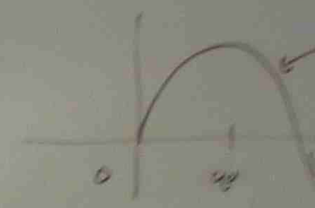
$\pi \rightarrow 180^\circ$

$\frac{\pi}{3} \rightarrow \frac{180}{3} = 60^\circ$

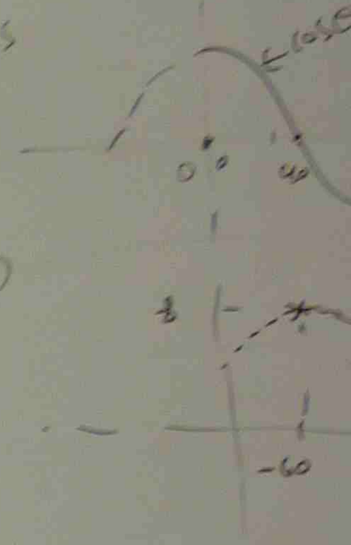
$8\cos(\theta - \pi/3) = 8\cos(\theta - 60^\circ)$

↑ AMPLITUDE

SINE

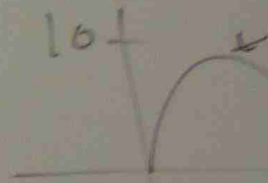


COS

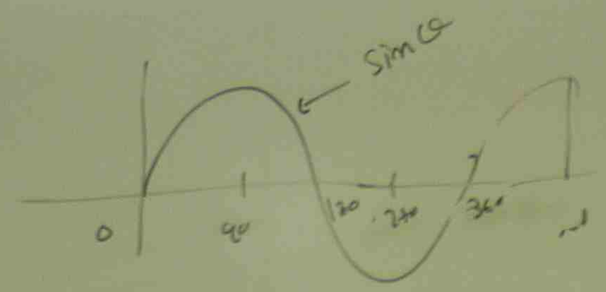


(d) $\pi = 180^\circ$

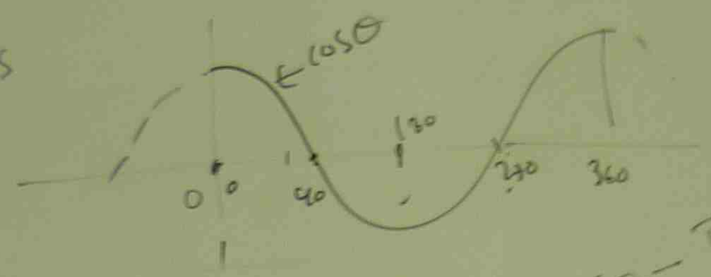
$\frac{\pi}{4} \Rightarrow \frac{180}{4} =$



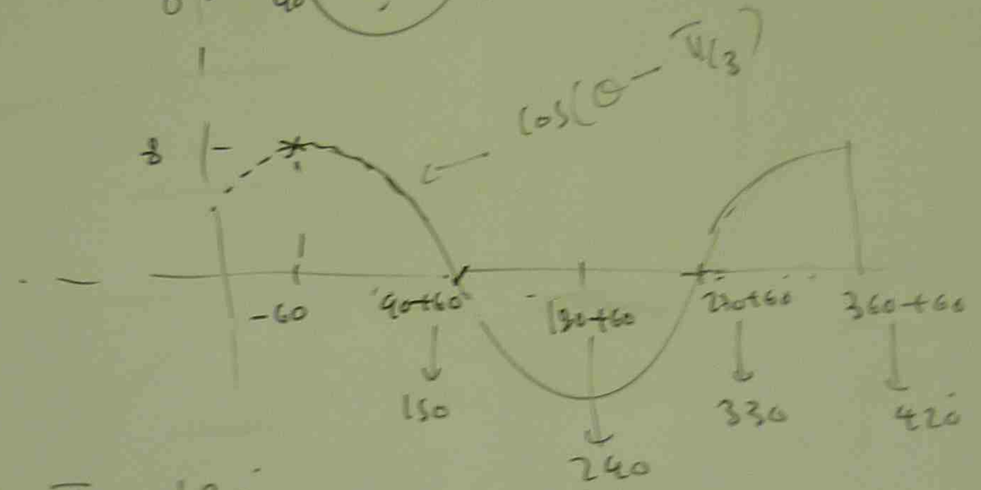
SINE



COS

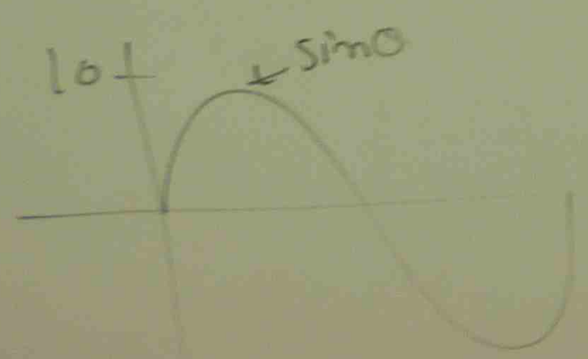


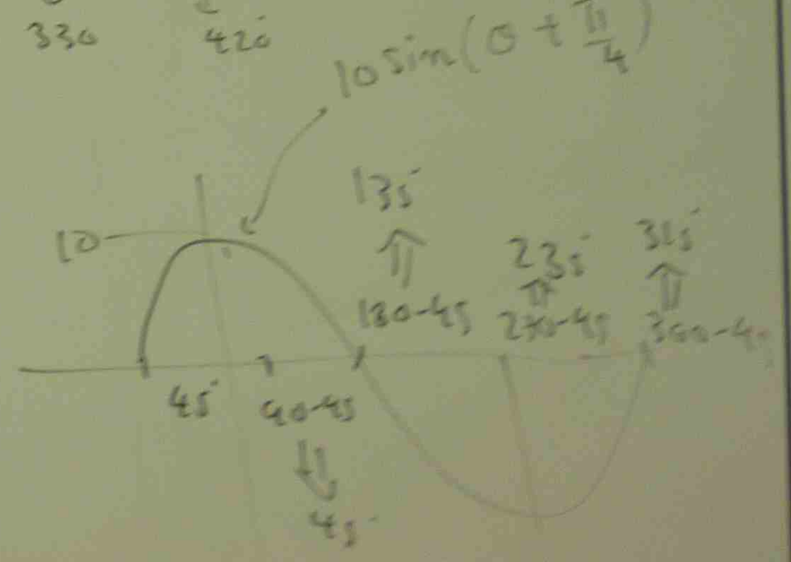
(c)



(d) $\pi = 180^\circ$

$\frac{\pi}{4} \Rightarrow \frac{180}{4} = 45^\circ$





=220

360+40=400°

ONVERSION

Supple

Sin θ =

cos θ =

tan θ =

180-θ

Sin (

cos(

tan(

EXERCISE

PLOT (a) $12\sin(\theta + 50^\circ)$

(b) $24\cos(\theta - 60^\circ)$

(c) $8\cos(\theta - \frac{\pi}{6})$

(d) $10\sin(\theta + \frac{\pi}{6})$

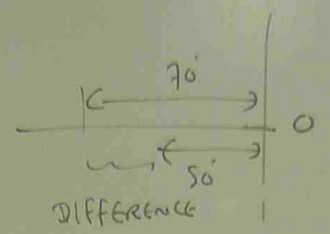
EX FIND THE PHASE ANGLE DIFFERENCE OF THE FOLLOWING TRIGO FUNCTIONS

(a) $7\sin(\theta + 70)$ & $12\sin(\theta + 50)$

(b) $6\cos(\theta + 10)$ & $24\cos(\theta - 60)$

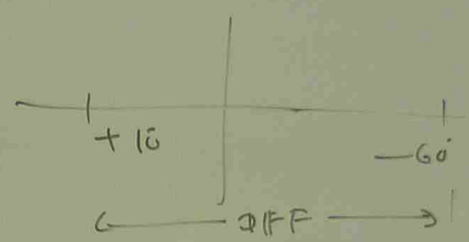
(c) $50\cos(\theta - 40)$ & $12\sin(\theta - 40)$

(a)



ANGLE DIFFERENCE = 70 − 50 = 20°

(b)



ANGLE DIFFERENCE = 10 + 60 = 70°

(c) $\sin(90 - \theta) = \cos\theta$

$\theta - 40$

$\sin(90 - (\theta - 40)) = \cos(\theta - 40)$

$\sin(90 - \theta + 40) = \cos(\theta - 40)$

$\sin(130 - \theta) = \cos(\theta - 40)$

$\sin -(\theta -$

$-\sin(\theta$

INSTEAD

$50($

$-$

$-$

FERENCE = 70 − 50 = 20°

$\sin(\theta - 130) = \cos(\theta - 40)$

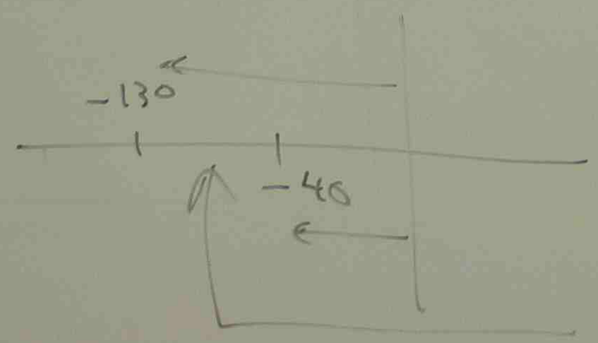
$-\sin(\theta - 130) = \cos(\theta - 40)$

INSTEAD OF $-\sin(\theta - 130)$ IT IS SUBSTITUTED

$50(-\sin(\theta - 130))$

$-50\sin(\theta - 130)$ & $12\sin(\theta - 40)$

ARE TO BE COMPARED.



ANGLE DIFFERENCE = 130 − 40 = 90°

+60 = 70°

s(θ − 40)

s(θ − 40)

− 40)

DEGREE ⟷ RADIAN CONVERSION

$180^\circ \longrightarrow \pi$ RADIAN

$360^\circ \longrightarrow 2\pi$ RADIAN

$90^\circ \longrightarrow \frac{\pi}{2}$ RADIAN

$270^\circ \longrightarrow \frac{3\pi}{2}$ RADIAN

Ex CONVERT THE FOLLOWING ANGLES INTO DEGREES.

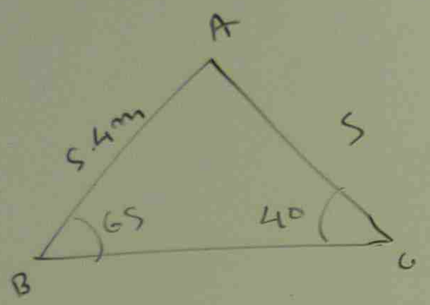
(a) $\frac{\pi}{4}$ (b) π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{10}$ (e) $\frac{5\pi}{12}$

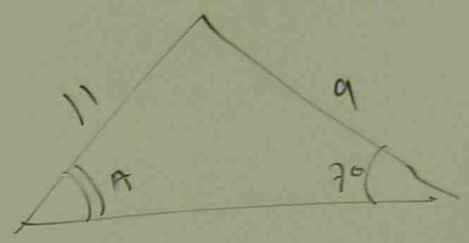
(a) $\frac{\pi}{4} \rightarrow \frac{180}{4} = 45^\circ$ (c) $\frac{\pi}{2} \rightarrow \frac{180}{2} = 90^\circ$ (e) $\frac{5\pi}{12} \rightarrow \frac{5 \times 180}{12} = 75^\circ$

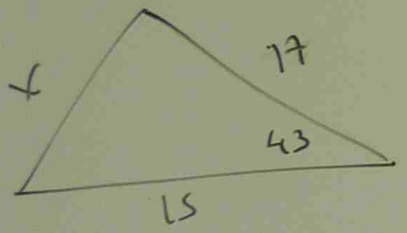
(b) $\pi \rightarrow 180^\circ$ (d) $\frac{\pi}{10} \rightarrow \frac{180}{10} = 18^\circ$

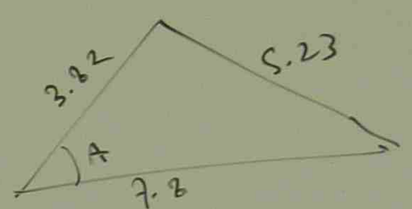
EXERCISE

IN EACH TRIANGLE BELOW, FIND THE LENGTH OF THE SIDES LABELLED









A

3.8 × 7.24 cos 80

34.7

mm

PLOTTING THE TRIGONOMETRIC FUNCTION GRAPH

$A \sin(\omega t - \phi)$ ← FORMAT

EX SKETCH THE CURVE $5.37 \sin(136 \times 10^3 t - 0.455)$ mA

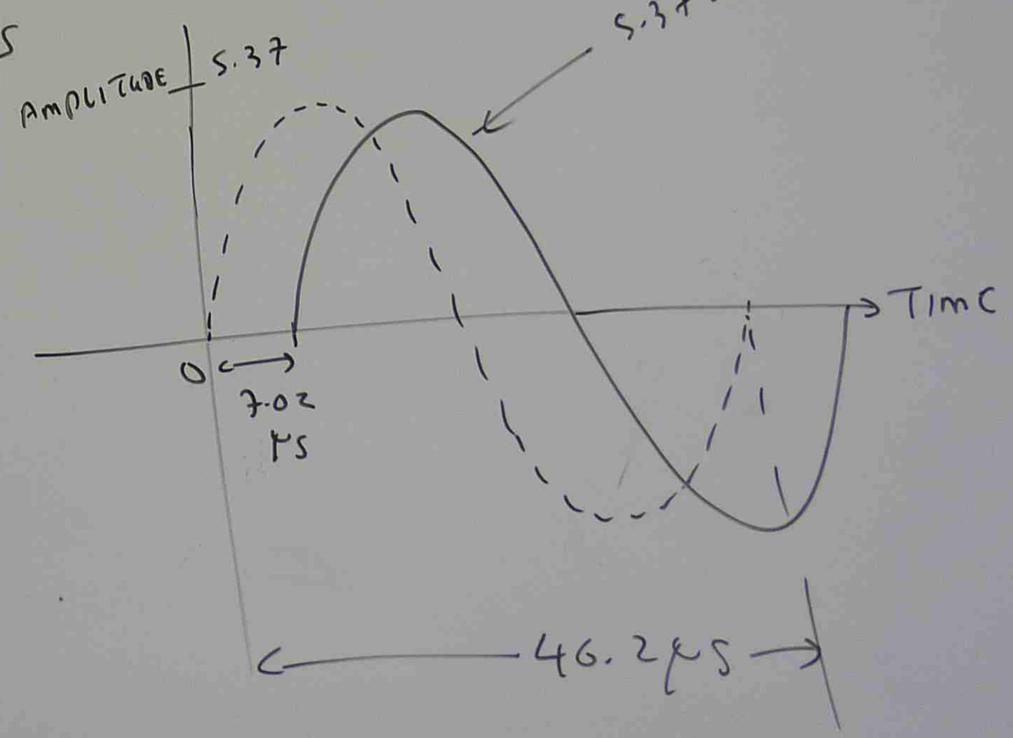
(ω = 136×10^3, ϕ = 0.455)

$$\text{PERIOD} = T = \frac{2\pi}{\omega}$$

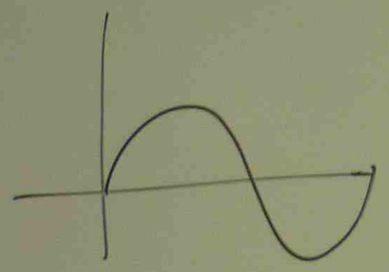
$$= \frac{2 \times 3.1416}{136 \times 10^3} = 46.2\ \mu s$$

$$\text{PHASE SHIFT} = \frac{\phi}{\omega}$$

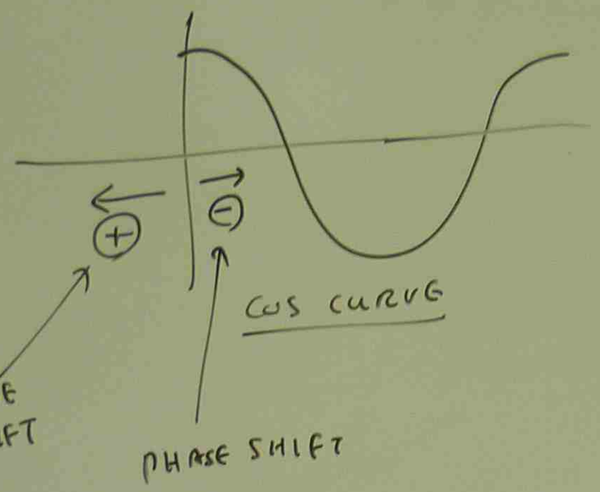
$$= \frac{0.455}{136 \times 10^3} = 7.02\ \mu s$$

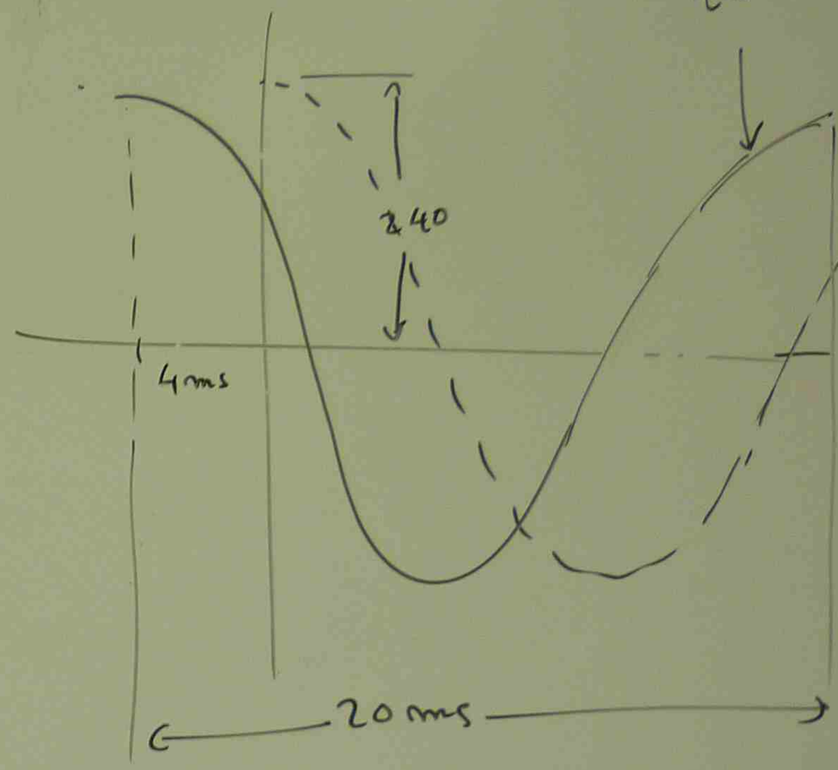


Ex SKETCH $V = 240 \cos(314t + 73)$ V



SINE CURVE





$$T = \frac{2\pi}{\omega} = \frac{2 \times 3.1416}{314} = 20\,\text{ms}$$

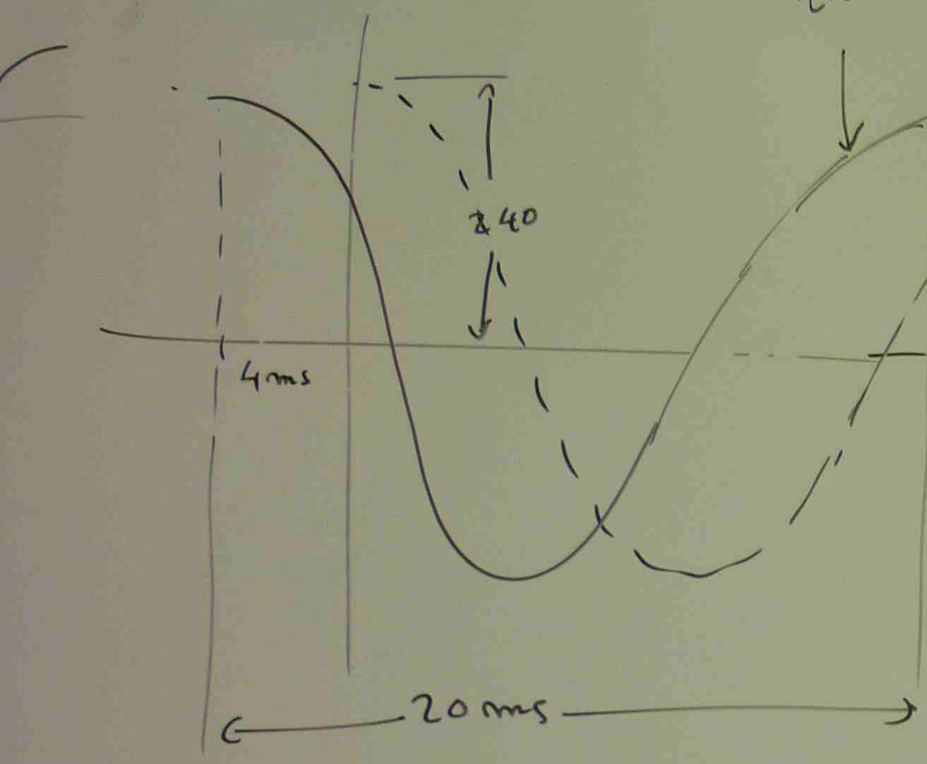
$$\text{PHASE SHIFT} = \frac{\phi}{\omega} = \frac{\phi\,(\text{RADIAN})}{\omega}$$

$180^\circ \longrightarrow \pi$ RADIAN

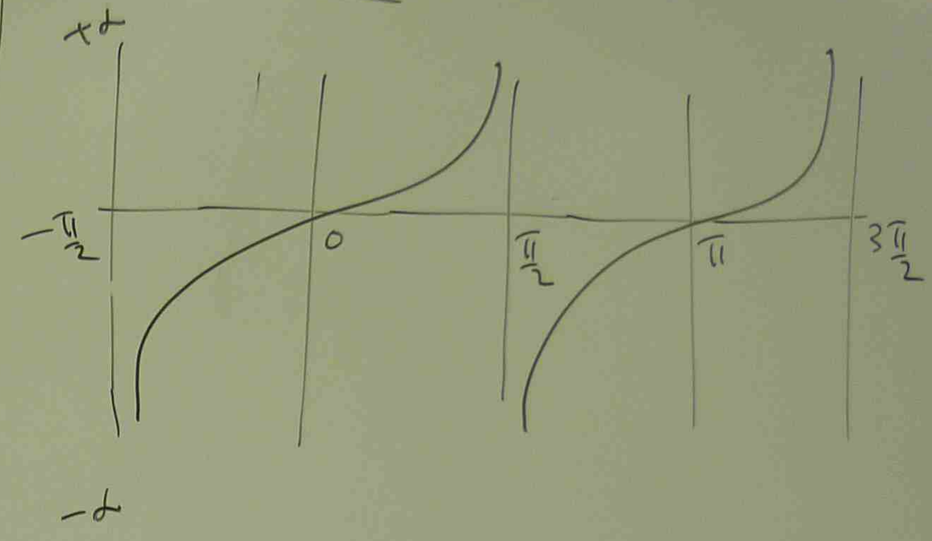
$73^\circ \longrightarrow ? = \frac{3.1416 \times 73}{180} = 1.274$ RAD

$$\text{PHASE SHIFT} = \frac{\phi}{\omega} = \frac{1.274}{314} = 0.004 = 4\,\text{ms}$$

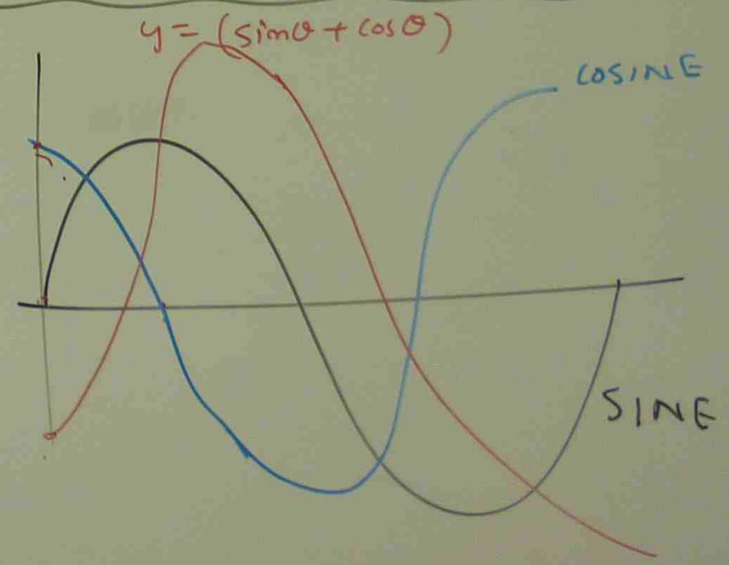
$240\cos(3(4t+73)$ V



TANGENT GRAPH



ADDING SINE AND COSINE FUNCTIONS



Ex FIND Ci

FIND TH

(a) $f = 50$

$T = \frac{1}{f}$

$f = 50\text{Hz} =$

(1 cycle =

ω

$\omega =$

Ex FIND (i) PERIOD & ANGULAR VELOCITY of (a) 50 HZ (b) 65 KHZ (c) 28 MHZ

FIND THE FREQUENCY AND ANGULAR VELOCITY of

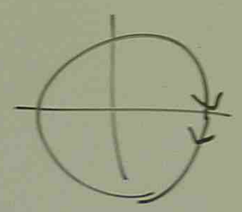
(a) 2 sec

(a) $f = 50\,HZ$

$$T = \frac{1}{f} = \frac{1}{50} = 0.02 \text{ sec}$$

$f = 50HZ = 50$ cycle/sec

(1 cycle $= 2\pi$ RADIAN)



$\omega = 50 \times 2\pi$ rad/s

$\omega = 314$ rad/s

(b) $f = 65KHZ$

$$T = \frac{1}{f} = \frac{1}{65 \times 10^3} = 0.065 \text{ sec}$$

$= 65$ ms

$f = 65KHZ = 65000$ HZ

$= 65000$ cycle/sec

$\omega = 65000 \times 2\pi$ rad/sec

$= 408408$ rad/sec.

(c) $f = 28MHZ$

$$T = \frac{1}{f} = \frac{1}{28 \times 10^6} = 0.0357 \mu s$$

$\omega = 28 \times 10^6$

$= 28 \times 10^6 \times$

$= 175.9$

(b) 2 sec

$T =$

$f = \frac{1}{T}$

ity of (a) 50 Hz (b) 65 kHz (c) 28 MHz

velocity of

(a) 2 sec

(b) $f = 65\text{kHz}$

$$T = \frac{1}{f} = \frac{1}{65\times10^3} = 0.065 \text{ sec}$$

$$= 65 \text{ ms}$$

$f = 65\text{kHz} = 65000 \text{ Hz}$

$= 65000 \text{ cycle/sec}$

$\omega = 65000 \times 2\pi \text{ rad/sec}$

$= 408408 \text{ rad/sec}.$

(c) $f = 28\text{MHz}$

$$T = \frac{1}{f} = \frac{1}{28\times10^6} = 0.0357 \ \mu s$$

$\omega = 28\times10^6 \text{ cycle/sec}$

$= 28\times10^6 \times 2\pi \text{ rad/sec}$

$= 175.9\times10^6 \text{ rad/sec}$

(b) 2 sec

$T = 2 \text{ sec}$

$$f = \frac{1}{T} = \frac{1}{2} = 0.5 \text{ Hz}$$

$= 0.5 \text{ cycle/sec}$

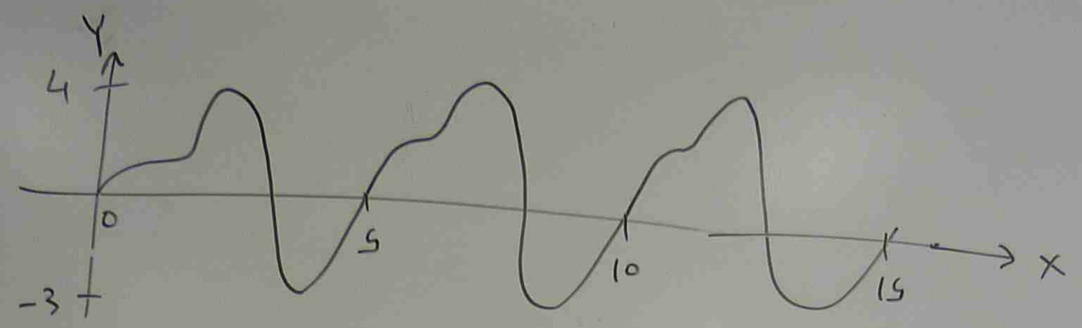
$= 0.5\times2\pi \text{ rad/sec}$

$= 0.5\times2\times3.1416 \text{ rad/sec}$

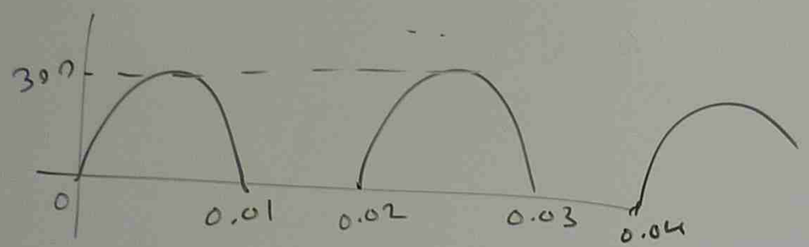
$= 3.1416 \text{ rad/sec}.$

Ex CALCULATE
(i) PERIOD
(ii) AMPLITUDE
OF THE GIVEN
CURVES

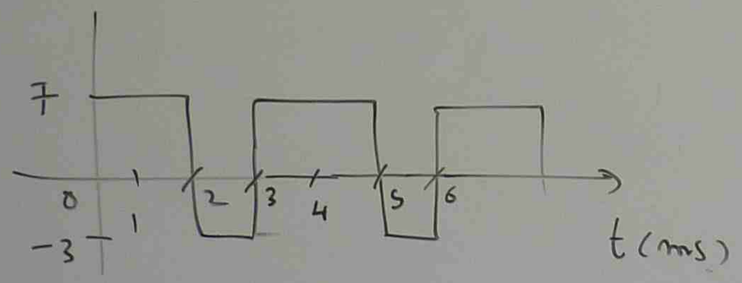
(a)



(b)



(c)



(a) PERIOD = 5 UNIT

AMPLITUDE = 4 + 3 = 7 UNIT

(b) PERIOD = 0.02

AMPLITUDE = 300

(c) PERIOD = 3 UNIT

AMPLITUDE = 7 + 3 = 10 UNIT

Ex

(a)

(b)

ERIOD = 5 UNIT

ITUDE = 4+3 = 7 UNIT

= 0.02

E = 300

= 3 UNIT

E. 7+3 = 10 UNIT

Ex plot (a) $y = 7\sin 2\theta$

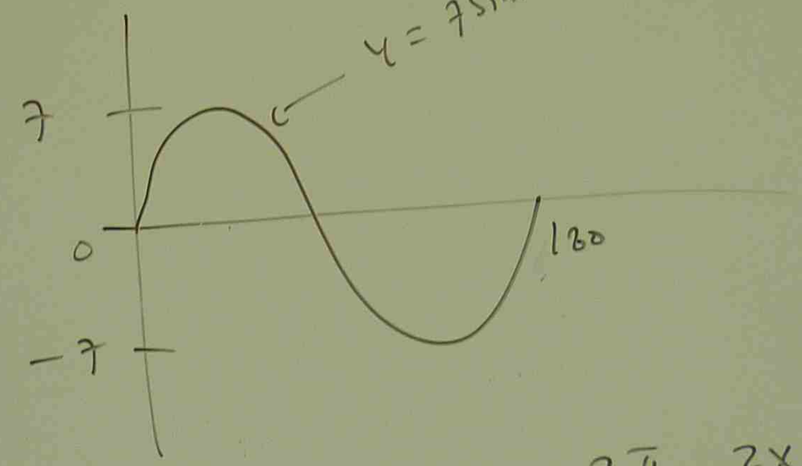
(b) $y = 220 \sin 0.4\theta$

(a) $y = 7 \sin 2\theta$

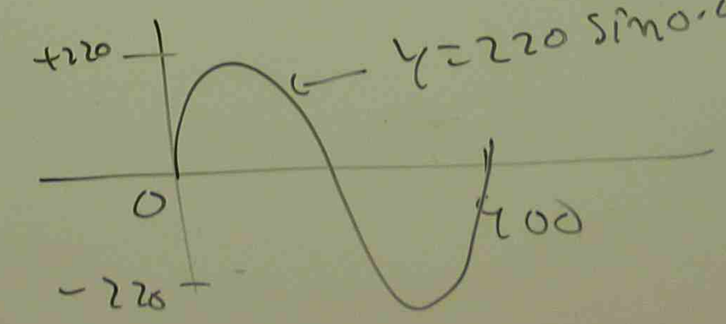
7 → HALF OF AMPLITUDE

2 → ω

PERIOD $= \frac{2\pi}{\omega} = \frac{2\times 180}{2} = 180$

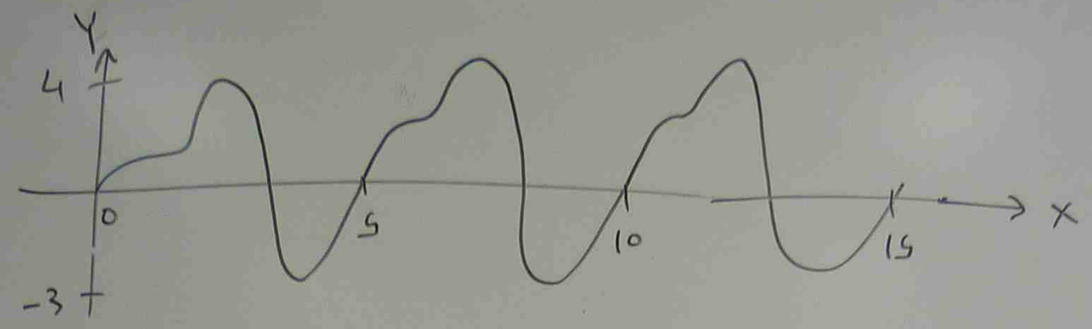


(b) $y = 220 \sin 0.4\theta \rightarrow$ PERIOD $= \frac{2\pi}{\omega} = \frac{2\times 180}{0.4} = 900$

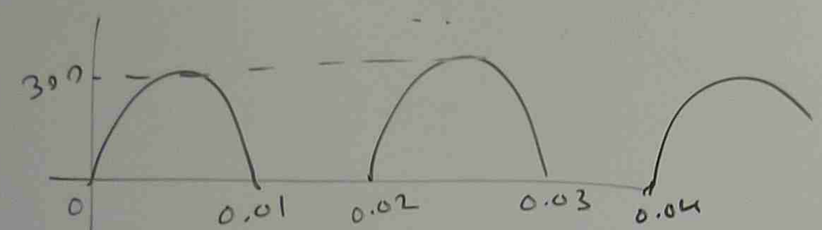


Ex CALCULATE
(i) PERIOD
(ii) AMPLITUDE
OF THE GIVEN
CURVES

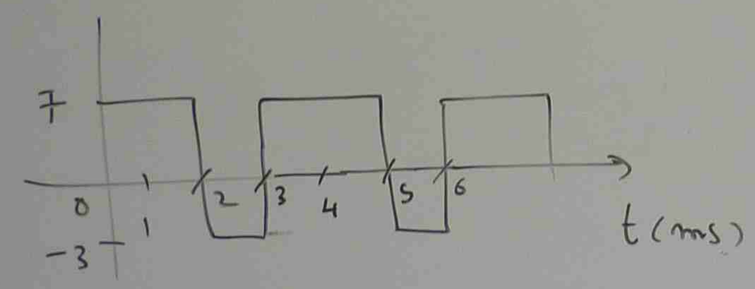
(a)



(b)



(c)



(a) PERIOD = 5 UNIT

AMPLITUDE = 4 + 3 = 7 UNIT

(b) PERIOD = 0.02

AMPLITUDE = 300

(c) PERIOD = 3 UNIT

AMPLITUDE = 7 + 3 = 10 UNIT

Ex

(a)

(a) PERIOD = 5 UNIT

AMPLITUDE = 4 + 3 = 7 UNIT

PERIOD = 0.02

AMPLITUDE = 300

PERIOD = 3 UNIT

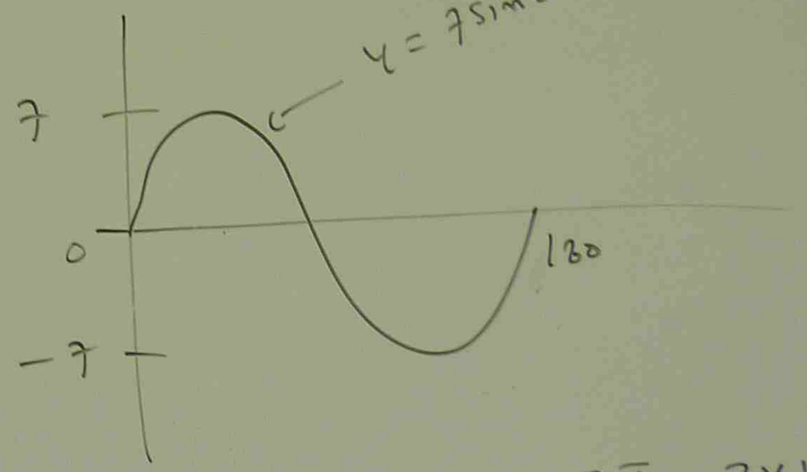
AMPLITUDE = 7 + 3 = 10 UNIT

<u>EX</u> plot (a) $y = 7 \sin 2\theta$

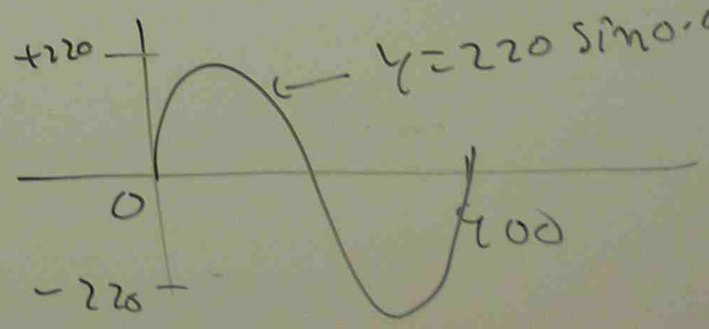
(b) $y = 220 \sin 0.4\theta$

(a) $y = \underset{\text{HALF OF AMPLITUDE}}{7} \sin \underset{\omega}{2}\theta$

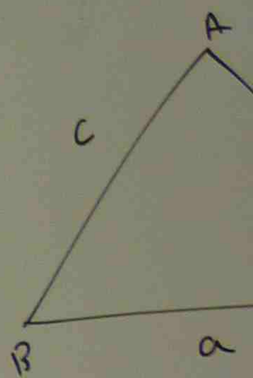
$\text{PERIOD} = \frac{2\pi}{\omega} = \frac{2 \times 180}{2} = 180$



(b) $y = 220 \sin 0.4\theta \rightarrow \text{PERIOD} = \frac{2\pi}{\omega} = \frac{2 \times 180}{0.4} = 900$



<u>TRIGOMETRY</u>



$\frac{a}{\sin A} = \frac{b}{\sin B}$

$a^2 = b^2 + c^2$

$b^2 = c^2 + a^2$

$c^2 = a^2 + b^2$

$= \frac{2\times180}{2} = 180$

$m\angle O$

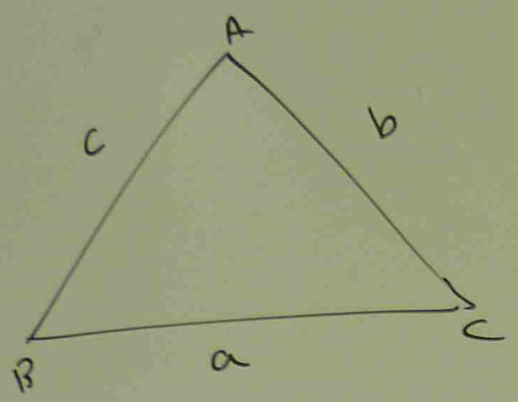
180

$D = \frac{2\pi}{\omega} = \frac{2\times180}{0.4} = 900$

$y = 220\sin 0.40$

900

TRIGOMETRY AND OBLIQUE ANGLES



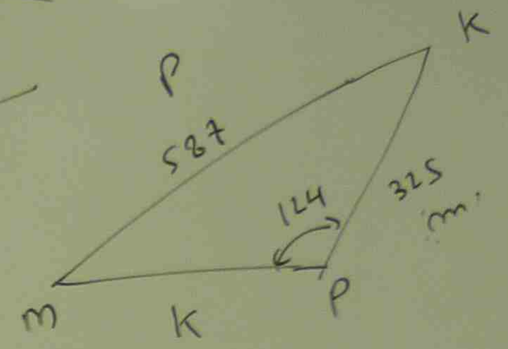
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = c^2 + a^2 - 2ca\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

EX



CALCULATE ANGLE M

$$\frac{P}{\sin P} = \frac{m}{\sin M}$$

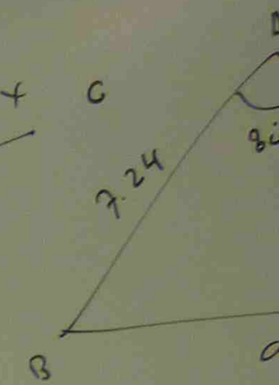
$$\frac{587}{\sin 124} = \frac{325}{\sin m}$$

$$\sin M = \frac{325\sin 124}{587}$$

$$\sin M = 0.459$$

$$m = \sin^{-1} 0.459 = 27.3^\circ$$

EX C



FIND 'a

$a^2 = b^2 + c^2$

$a^2 = 13.8^2 +$

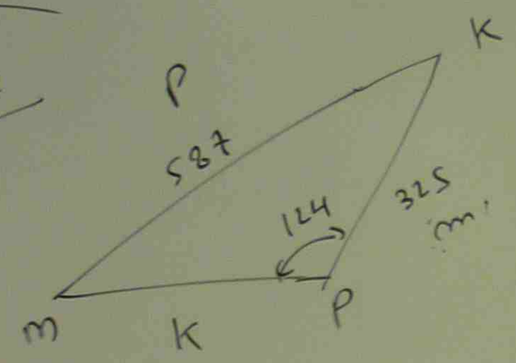
$a^2 = 524$

$a^2 = 208$

$a = \sqrt{20}$

OBLIQUE ANGLES

EX



CALCULATE ANGLE M.

$$\frac{P}{\sin P} = \frac{m}{\sin M}$$

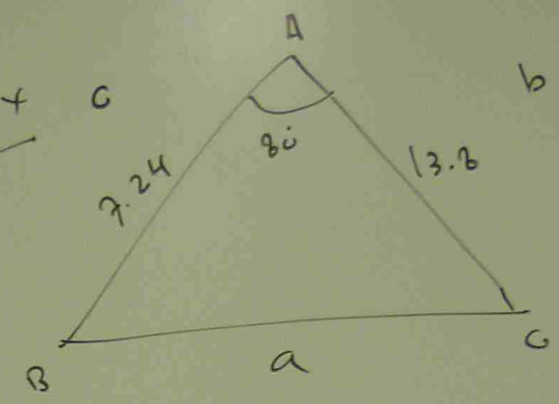
$$\frac{587}{\sin 124} = \frac{325}{\sin M}$$

$$\sin M = \frac{325 \sin 124}{587}$$

$$\sin M = 0.459$$

$$m = \sin^{-1} 0.459 = 27.3°$$

EX c



FIND "a"

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 13.8^2 + 7.24^2 - 2 \times 13.8 \times 7.24 \cos 80$$

$$a^2 = 52.42 + 190.4 - 34.7$$

$$a^2 = 208.2$$

$$a = \sqrt{208.2} = 14.4 \text{ mm}$$

EXERCISE

IN EACH T

